سارینا ستاره — کابین ۱۳

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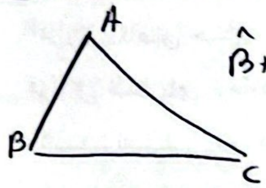
①



$\hat{B}+\hat{C}=۱۲۰^\circ \Rightarrow \hat{A}=۶۰^\circ$

$\dfrac{۲۰}{\frac{\sqrt{۳}}{۲}}=\dfrac{\frac{۲۰\sqrt{۶}}{۳}}{\sin\hat{B}} \Rightarrow \sin\hat{B}=\dfrac{۱۰\sqrt{۲}}{۲۰}=\dfrac{\sqrt{۲}}{۲} \rightarrow \hat{B}=۴۵$ یا ۱۳۵° (غ ق ق)

$\hat{C}=۷۵$

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②

$۱۸۰-\underbrace{(۶۰+۷۵)}_{۱۳۵}=۴۵ \qquad \dfrac{۲\sqrt{۲}}{\frac{\sqrt{۲}}{۲}}=\dfrac{AC}{\frac{\sqrt{۳}}{۲}} \Rightarrow AC=۲\sqrt{۳}$

$\sin ۶۰=\dfrac{\sqrt{۳}}{۲}=\dfrac{x}{۲\sqrt{۳}} \Rightarrow \boxed{x=۳}$

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③

$\dfrac{\sin C}{c}=\dfrac{\sin B}{b^۲-۲} \Rightarrow b^۲-۲=b \Rightarrow b^۲-b-۲=۰ \Rightarrow (b-۲)(b+۱) \rightarrow b=-۱,\ \boxed{b=۲}$

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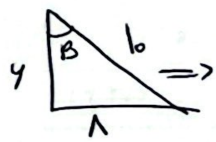
④

$۴۴۱+۴۰۰-۲\times۲۰\times۲۱\times\cos B=۱۶۹$

$۸۴۰\cos B=۶۷۲ \Rightarrow \cos B=\dfrac{۶۷۲}{۸۴۰}=$ ۶۸



$\Rightarrow \boxed{\sin B=\dfrac{۶}{۱۰}}$

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⑤

$a^۲=\underbrace{۱۶+۴(۱+۳+۲\sqrt{۳})}_{۳۲+۸\sqrt{۳}}\underbrace{-۲\times\frac{۱}{۲}\times۴\times۲\times(۱+\sqrt{۳})}_{-۸-۸\sqrt{۳}}=۲۴ \Rightarrow \boxed{a=۲\sqrt{۶}}$ (الف)

$\dfrac{۲\sqrt{۶}}{\frac{\sqrt{۳}}{۲}}=\dfrac{۴}{\sin B} \Rightarrow \sin B=\dfrac{\sqrt{۲}}{۲} \Rightarrow B=۴۵$ یا ۱۳۵ (غ ق ق) $\rightarrow$ (ب)

$C=۷۵$

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⑥

$\triangle ABC:\ ۴=۴+۱-۱\times۲\times۲\times\cos\alpha \Rightarrow ۴\cos\alpha=۱ \Rightarrow \cos\alpha=\frac{۱}{۴}$

$ADE \Rightarrow DE^۲=\underbrace{۱۶+۳۶}_{۵۲}\underbrace{-۲\times۴\times۶\times\frac{۱}{۴}}_{۱۲}=۴۰ \Rightarrow \boxed{DE=۲\sqrt{۱۰}}$

$CB^2 = \frac{64+25}{89} - \frac{2\times\frac{1}{2}\times 5\times 8}{40} = 49 \Rightarrow CB = 7$ (الف)

(٧)

$S_{ABC} = \frac{1}{2}\times 5\times 8\times \frac{\sqrt{3}}{2} = 10\sqrt{3}$

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(٨)

$\frac{b^2+c^2+2bc-a^2}{-b^2c^2+2bc\cos\alpha} = \frac{2(1+\cos\alpha)bc}{(\cos\alpha+1)bc} = 2$

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(٩)

$a^2+b^2-c^2 = a^2+ba^2-ca^2$

$(b-c)(b^2+c^2+bc) = a^2(b-c) \Rightarrow b^2+c^2+bc = a^2$

$b^2+c^2-2bc\cos A = a^2$

$\Rightarrow -2\cos A = 1 \Rightarrow \cos A = -\frac{1}{2}$

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(١٠)

$(1+2\cos\theta)(2-\cos\theta)\times\frac{1}{2}\times\frac{1}{2} = 2$

$2+8\cos\theta-2\cos^2\theta = 8$

$2\cos^2\theta-8\cos\theta+5=0 \Rightarrow (2\cos\theta-5)(\cos\theta-1)=0$

$\cos\theta = 1$ ✓ قبول

$\cos\theta = \frac{5}{2}$ ✗ غیرقابل قبول

$a^2 = 4+16-\frac{2\sqrt{3}}{2}\times\frac{1}{2}\times 2\times 4 = 20-4\sqrt{3}$