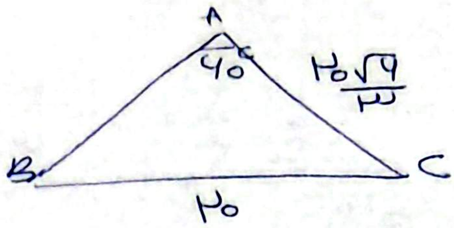
[illegible] دوازدهم [illegible]



۱- $\frac{a}{\sin\hat{A}}=\frac{b}{\sin\hat{B}} \to \frac{20}{\frac{\sqrt{3}}{2}}=\frac{\frac{20\sqrt{6}}{3}}{\sin B}$

$\sin B=\frac{\sqrt{18}}{6}=\frac{\sqrt{3}}{2} \Rightarrow \hat{B}=30^\circ \quad \hat{C}=90^\circ$

۲-

$\frac{AC}{\sin\hat{D}}=\frac{CD}{\sin\hat{A}} \to \frac{AC}{\frac{\sqrt{3}}{2}}=\frac{2\sqrt{2}}{\frac{\sqrt{2}}{2}}$

$180-(75+60)=45$

$\boxed{AC=2\sqrt{3}}$

$BC=\frac{\sqrt{3}}{2}AC=\frac{\sqrt{3}}{2}\times 2\sqrt{3}=3$

۳-

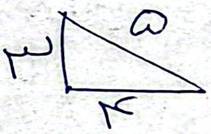
$\frac{a}{\sin\hat{A}}=\frac{b}{\sin\hat{B}}=\frac{c}{\sin\hat{C}} \to c=\frac{b\sin\hat{C}}{\sin\hat{B}} \to (b^2-2)\frac{\sin\hat{C}}{\sin\hat{B}}=\frac{b\sin\hat{C}}{\sin\hat{B}}$

$\to b^2-b-2=0 \to (b-2)(b+1)=0 \to b=2$

۴-

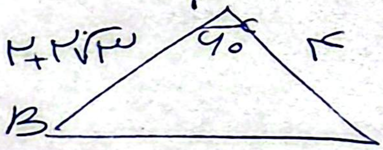
$AC^2=AB^2+BC^2-2(AB)(BC)(\cos\hat{B}) \Rightarrow 169=\underbrace{400+441}_{841}-2(20)(21)\cos\hat{B}$

$\to \cos\hat{B}=\frac{4}{5}$



$\to \sin\hat{B}=\frac{3}{5}$

۵-



$a^2=b^2+c^2-2bc\cos\hat{A}$

$\to a^2=\underbrace{16+4}_{20}+12+8\sqrt{3}-2(4)(2+2\sqrt{3})\left(\frac{1}{2}\right)$

$32+8\sqrt{3}-8-8\sqrt{3}=24 \Rightarrow a=\sqrt{24}=2\sqrt{6}$

$\frac{a}{\sin\hat{A}}=\frac{b}{\sin\hat{B}} \Rightarrow \frac{2\sqrt{6}}{\frac{\sqrt{3}}{2}}=\frac{4}{\sin\hat{B}} \Rightarrow \sin\hat{B}=\frac{\sqrt{2}}{2}$ (۶

$\Rightarrow \hat{B}=45^\circ$

$\hat{C}=180-(\hat{A}+\hat{B})=180-(105)=75^\circ$

$\cos\alpha = \frac{1}{4}$

$DF^2 = AD^2 + AE^2 - 2(AD)(AE)\cos\alpha = \underbrace{16+36}_{52} - \underbrace{2(4)(6)\left(\frac{1}{4}\right)}_{12}$

$= 40 \rightarrow DE = 2\sqrt{10}$

-7

$BC^2 = AC^2 + AB^2 - 2(AC)(AB)\cos\hat{A} =$

$\underbrace{25+64}_{89} - \underbrace{2(5)(8)\frac{1}{2}}_{40} = 49 \rightarrow BC = 7$ (أ)

(ب) $S_{ABC} = \frac{1}{2} \times AC \times AB \times \sin\hat{A} = \frac{1}{2} \times 8 \times 5 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$

-8

$a^2 = b^2 + c^2 - 2bc\cos\hat{A}$

$\rightarrow b^2 + c^2 - a^2 = 2bc\cos\hat{A}$ $\rightarrow \frac{2bc\cos\hat{A} + 2bc}{bc(\cos\hat{A}+1)} = 2$ (1)

-9

$a^3 + b^3 - c^3 = a^3 + a^2b - a^2c \rightarrow (b-c)(b^2+c^2+bc) = a^2(b-c)$

$\rightarrow a^2 = b^2 + c^2 + bc = b^2 + c^2 - 2bc\cos\hat{A}$

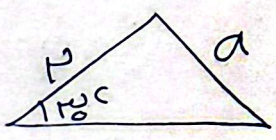
$\rightarrow \cos\hat{A} = -\frac{1}{2}$

-10

$\frac{1}{2} \times \frac{1}{2} \times (3\cos\theta)(1+2\cos\theta) = 2$

$\rightarrow 2 + 9\cos\theta - \cos\theta - 3\cos^2\theta = 8$

$\rightarrow 3\cos^2\theta - 8\cos\theta + 6 = 0$ $\cos\theta = \frac{8 \pm 2}{6} = \frac{6}{3}$ (1)



$a^2 = 4 + 16 - 2(2)(4)\left(\frac{\sqrt{3}}{2}\right)$

$= 20 - 8\sqrt{3}$