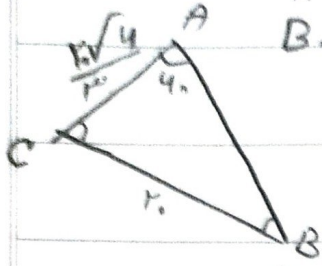
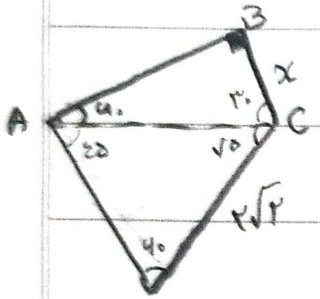


$\hat{B}+\hat{C}+\hat{A}=180 \xrightarrow{\hat{B}+\hat{C}=135} \hat{A}=45^\circ$

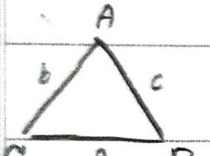
$\Rightarrow \frac{BC}{\sin A}=\frac{AC}{\sin B} \rightarrow \frac{20}{\frac{\sqrt{2}}{2}}=\frac{\frac{20\sqrt{6}}{3}}{\sin B}$ -۱

$\sin B=\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}\sqrt{3}}{2}=\frac{\sqrt{3}}{2} \rightarrow \hat{B}=60^\circ, C=75^\circ$



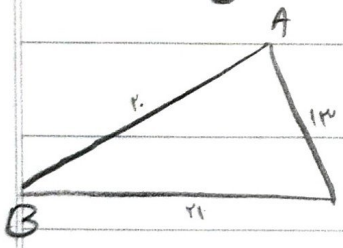
$\frac{\frac{\sqrt{2}}{2}}{2\sqrt{2}}=\frac{\frac{\sqrt{3}}{2}}{AC} \Rightarrow AC=2\sqrt{2}$ -۲

$\sin 45=\frac{\sqrt{2}}{2}=\frac{x}{2\sqrt{2}} \Rightarrow x=2$



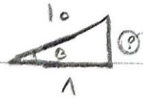
$\frac{c}{\sin\hat{C}}=\frac{b}{\sin B} \Rightarrow \frac{\sin\hat{C}}{\sin\hat{B}}=\frac{c}{b}$ -۳

$\Rightarrow \frac{(b^2-2)c}{b}=c \Rightarrow b^2-2=b \rightarrow b^2-b-2=0 \begin{cases} b=-1 \times \\ b=2 \checkmark \end{cases}$

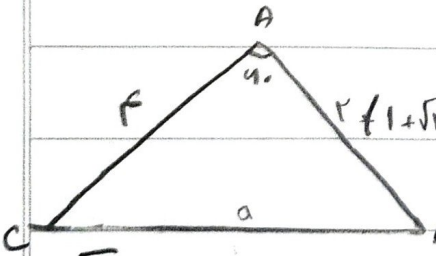


$13^2=20^2+21^2-2(21)(20)(\cos\hat{B})$ -۴

$47=840\times\cos B \rightarrow \cos\hat{B}=0.8$



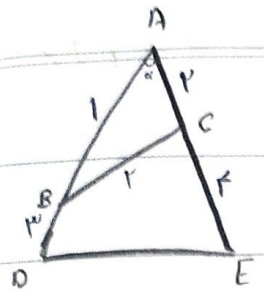
$\rightarrow \sin\hat{B}=0.6$



$a^2=16+16+8\sqrt{2}-8\sqrt{2}-8$ (ت الف

$a=\sqrt{24}=2\sqrt{6}$

$\frac{2\sqrt{6}}{\frac{\sqrt{2}}{2}}=\frac{4}{\sin B} \Rightarrow \sin B=\frac{\sqrt{2}}{2} \rightarrow B=45^\circ, C=75^\circ$ (ب

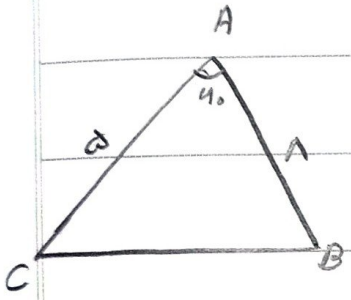


۹- $\triangle ABC: ۲^۲ = ۲^۲ + ۱^۲ - ۲(۱)(۲)(\cos\alpha)$

$\Rightarrow \cos\alpha = \frac{۱}{۴}$

$\triangle ADE: DE^۲ = ۱۶ + ۳۶ - ۲(۴)(۶)(\frac{۱}{۴}) = ۴۸$

$\Rightarrow DE = ۴\sqrt{۱۰}$



۷- الف) $BC^۲ = ۵^۲ + ۸^۲ - ۲(۵)(۸)(\frac{۱}{۲}) = ۴۹ \Rightarrow BC = ۷$

ب) $S = \frac{۱}{۲} \times ۸ \times ۵ \times \sin ۶۰ = ۱۰\sqrt{۳}$ ($\sin ۶۰ = \frac{\sqrt{۳}}{۲}$)

۸- $\frac{b^۲ + c^۲ + ۲bc - a^۲}{bc(\cos\hat{A} + ۱)} \Rightarrow \frac{۲bc(\cos\hat{A} + ۱)}{bc(\cos\hat{A} + ۱)} = ۲$

$a^۲ = b^۲ + c^۲ - ۲bc\cos\hat{A}$

۹- $a^۳ + b^۳ - c^۳ = a^۳ + a^۲b - a^۲c \Rightarrow (b-c)(b^۲ + bc + c^۲) = a^۲(b-c)$

$\Rightarrow b^۲ + c^۲ + bc = a^۲ \Rightarrow bc = -۲bc\cos A$

$b^۲ + c^۲ - ۲bc\cos A = a^۲ \Rightarrow \cos A = -\frac{۱}{۲}$

۱۰- $S = (۱ + ۲\cos\theta)(۳ - \cos\theta) \times \frac{۱}{۲} \times \frac{۱}{۲} = ۲$

$\Rightarrow -۲\cos^۲\theta + ۸\cos\theta + ۳ = ۸ \Rightarrow \begin{cases} \cos\theta = ۱ \checkmark \\ \cos\theta = \frac{۵}{۲} \times \end{cases}$

$a^۲ = ۲^۲ + ۴^۲ - ۲(۲)(۴)(\frac{\sqrt{۳}}{۲}) = ۴ + ۱۶ - ۸\sqrt{۳} = ۲۰ - ۸\sqrt{۳}$

SCBO