

۲. آفرین

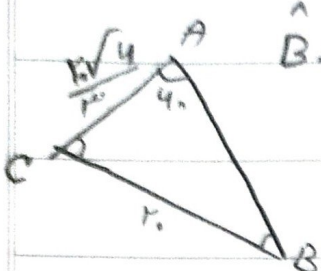
۱۱

کتاب ۱۱

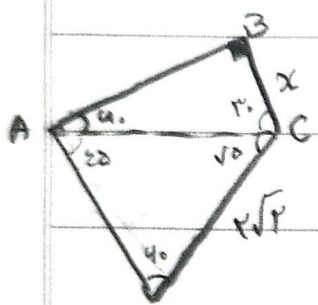
موسسه

$\hat{B} + \hat{C} + \hat{A} = 180^\circ \xrightarrow{\hat{B} + \hat{C} = 140^\circ} \hat{A} = 40^\circ$

$\Rightarrow \frac{BC}{\sin A} = \frac{AC}{\sin B} \rightarrow \frac{r_0}{\frac{\sqrt{r}}{r}} = \frac{r_0 \sqrt{4}}{\sin B}$ - ۱

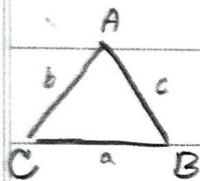


$\sin B = \frac{\sqrt{r}}{r} \times \frac{\sqrt{r} \sqrt{r}}{r} = \frac{\sqrt{r}}{r} \rightarrow \hat{B} = 45^\circ, \hat{C} = 95^\circ$



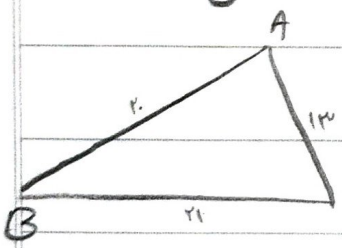
$\frac{\frac{\sqrt{r}}{r}}{r\sqrt{r}} = \frac{\frac{\sqrt{r}}{r}}{AC} \Rightarrow AC = r\sqrt{r}$ - ۲

$\sin 40^\circ = \frac{\sqrt{r}}{r} = \frac{x}{r\sqrt{r}} \Rightarrow x = r$



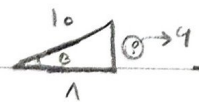
$\frac{c}{\sin \hat{C}} = \frac{b}{\sin \hat{B}} \Rightarrow \frac{\sin \hat{C}}{\sin \hat{B}} = \frac{c}{b}$ - ۳

$\Rightarrow \frac{(b-r)c}{b} = c \rightarrow b-r=b \rightarrow b-b-r=0 \Rightarrow \begin{cases} b=-r \\ b=r \end{cases}$

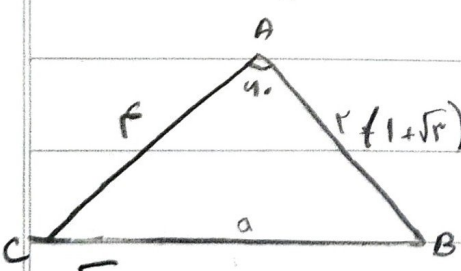


$r^2 = r_0^2 + r_1^2 - 2(r_0)(r_1)(\cos \hat{B})$

$4r^2 = 16 + 16 - 2(4)(4)(\cos \hat{B}) \rightarrow \cos \hat{B} = 0.1$



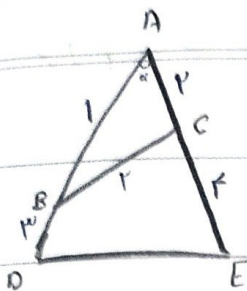
$\rightarrow \sin \hat{B} = 0.4$



$a^2 = 14 + 14 + 1\sqrt{r} - 1\sqrt{r} = 28$ (د انج)

$a = \sqrt{28} = r\sqrt{4}$

$\frac{r\sqrt{4}}{\frac{\sqrt{r}}{r}} = \frac{r}{\sin B} \Rightarrow \sin B = \frac{\sqrt{r}}{r} \rightarrow \hat{B} = 45^\circ, \hat{C} = 95^\circ$

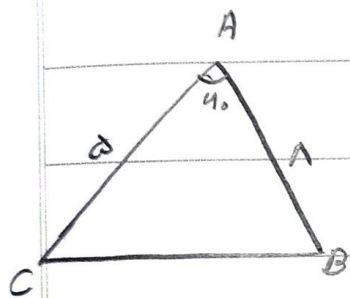


$$\Delta ABC: r^2 = 1^2 + r^2 - 2(1)(r)(\cos \alpha) \quad \text{--- 4}$$

$$\Rightarrow \cos \alpha = \frac{1}{r} \quad \text{--- 2}$$

$$\Delta ADE: DE^2 = 1^2 + r^2 - 2(1)(r)\left(\frac{1}{r}\right) = r^2 \quad \checkmark$$

$$\Rightarrow DE = r\sqrt{10} \quad \checkmark$$



$$BC^2 = a^2 + a^2 - 2(a)(a)\left(\frac{1}{r}\right) = 29 \rightarrow BC = \sqrt{29} \quad \checkmark \quad \text{الف}$$

$$S = \frac{1}{2} \times a \times a \times \sin \alpha_0 = 10\sqrt{10} \quad \checkmark \quad \text{ب}$$

$$\frac{b^2 + c^2 + rbc - a^2}{bc(\cos \hat{A} + 1)} \rightarrow \frac{rbc(\cos \hat{A} + 1)}{bc(\cos \hat{A} + 1)} = r \quad \checkmark \quad \text{--- 1}$$

$$a^2 = b^2 + c^2 - rbc \cos \hat{A}$$

$$a^2 + b^2 - c^2 = a^2 + a^2b - a^2c \rightarrow b \cdot c / (b^2 + bc + c^2) = a^2(b-c) \quad \text{--- 9}$$

$$\Rightarrow \left. \begin{aligned} b^2 + c^2 + bc &= a^2 \\ b^2 + c^2 - rbc \cos A &= a^2 \end{aligned} \right\} \Rightarrow bc = -rbc \cos A \quad \text{--- 10}$$

$$\cos A = -\frac{1}{r} \quad \checkmark \rightarrow \hat{A} = 120^\circ$$

$$S = (1 + r \cos \theta)(r - \cos \theta) \times \frac{1}{r} \times \frac{1}{r} = r \quad \text{--- 10}$$

$$\Rightarrow -r \cos \theta + 1 \cos \theta + r = 1 \Rightarrow \begin{cases} \cos \theta = 1 \quad \checkmark \\ \cos \theta = \frac{1}{r} \times \end{cases} \quad \text{--- 10}$$

$$a^2 = r^2 + r^2 - r(r)(1)\left(\frac{\sqrt{10}}{r}\right) = r^2 + 14 - 14\sqrt{10} = r_0 - 14\sqrt{10}$$