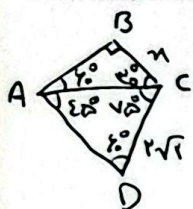


$BC = 20$ * $\hat{B} + \hat{C} = 120^\circ$ $AC = \frac{20\sqrt{2}}{3}$ $\hat{B}, \hat{C}?$

$$\frac{b}{\sin \hat{B}} = \frac{a}{\sin \hat{A}} = \frac{c}{\sin \hat{C}} \rightarrow \frac{20}{\sqrt{2}} = \frac{20\sqrt{2}}{\sin \hat{B}} \rightarrow \sin \hat{B} = \frac{\sqrt{2}}{2} \rightarrow \hat{B} = 45^\circ \text{ یا } 135^\circ$$

* طبق

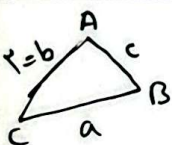
$\hat{C} = 120 - 45 = 75^\circ$



$$\frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}} = \frac{a}{\sin \hat{A}}$$

$$\frac{20}{\sin 45^\circ} = \frac{AC}{\sin 75^\circ} \rightarrow \frac{20}{\frac{\sqrt{2}}{2}} = \frac{AC}{\frac{\sqrt{6}+\sqrt{2}}{4}} \rightarrow AC = \frac{20\sqrt{2}}{3}$$

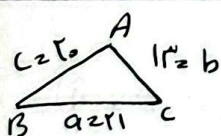
$$\frac{AC}{\sin 75^\circ} = \frac{20\sqrt{2}}{\sin 45^\circ} \rightarrow AC = 20\sqrt{2}$$



$$(b^2 - r^2) \frac{\sin \hat{C}}{\sin \hat{B}} = c$$

$$\frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}} \rightarrow \frac{\sin \hat{C}}{\sin \hat{B}} = \frac{c}{b} \rightarrow (b^2 - r^2) \times \frac{c}{b} = c \rightarrow b^2 - r^2 = b \rightarrow b^2 - b - r^2 = 0$$

$b = 10 \pm \sqrt{100 - r^2}$



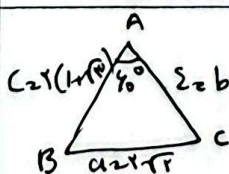
$$b^2 = a^2 + c^2 - 2ac \cos \hat{B}$$

$$149 = 221 - 2 \times 20 \times 20\sqrt{2} \times \cos \hat{B}$$

$$128 = -40\sqrt{2} \cos \hat{B}$$

$$\cos \hat{B} = -\frac{2}{\sqrt{2}}$$

$$\cos^2 \hat{B} + \sin^2 \hat{B} = 1 \rightarrow \frac{4}{2} + \sin^2 \hat{B} = 1 \rightarrow \sin \hat{B} = \frac{\sqrt{2}}{2}$$



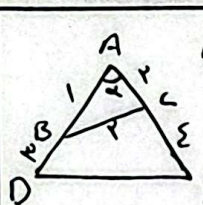
$$a^2 = b^2 + c^2 - 2bc \cos \hat{A}$$

$$a^2 = 14 + 14 + 14\sqrt{2} - 2 \times 2 \times 20\sqrt{2} \times \cos \hat{A} = 32 + 14\sqrt{2} - 40\sqrt{2} \cos \hat{A} = 22$$

$a = 2\sqrt{2}$

$$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}} \rightarrow \frac{2\sqrt{2}}{\sin \hat{A}} = \frac{20}{\sin 45^\circ} \rightarrow \sin \hat{A} = \frac{\sqrt{2}}{10} \rightarrow \hat{A} = 45^\circ \text{ یا } 135^\circ$$

$\hat{C} = 120 - (45 + 45) = 30^\circ$

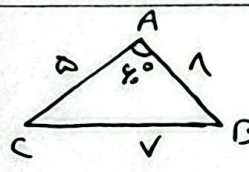


$$\triangle ABC \Rightarrow c^2 = b^2 + a^2 - 2ab \cos \alpha \Rightarrow \cos \alpha = \frac{1}{2}$$

$$\triangle ADE \Rightarrow DE^2 = AD^2 + AE^2 - 2 \times AD \times AE \times \cos \alpha$$

$$DE^2 = 1^2 + 2^2 - 2 \times 1 \times 2 \times \frac{1}{2} = 2^2 - 1^2 = 3$$

$$DE = \sqrt{3} = \boxed{1\sqrt{3}}$$



$$BC^2 = AC^2 + AB^2 - 2 \times AC \times AB \times \cos \hat{A}$$

$$BC^2 = 2^2 + 1^2 - 2 \times 2 \times 1 \times \frac{1}{2}$$

$$BC^2 = 4 + 1 - 2 = 3 \rightarrow BC = \sqrt{3}$$

$$S = \frac{1}{2} \times 2 \times 1 \times \frac{\sqrt{3}}{2} = \boxed{1\sqrt{3}}$$

$$\frac{(b+c)^2 - a^2}{bc(\cos \hat{A} + 1)} = \frac{b^2 + c^2 + 2bc - b^2 - c^2 + 2bc \cos \hat{A}}{bc(\cos \hat{A} + 1)} = \frac{2bc(\cos \hat{A} + 1)}{bc(\cos \hat{A} + 1)} = \boxed{2}$$

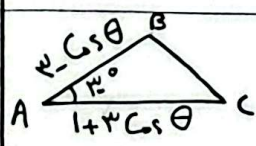
$$a^2 = b^2 + c^2 - 2bc \cos \hat{A}$$

$$\frac{a^2 + b^2 - c^2}{a+b-c} = a^2 \rightarrow b^2 + c^2 - 2bc \cos \hat{A}$$

$$a^2 + b^2 - c^2 = ab^2 + ac^2 - 2abc \cos \hat{A} + b^2 + c^2 - 2b^2c \cos \hat{A} - cb^2 + 2bc^2 \cos \hat{A}$$

$$a^2 = a(b^2 + c^2 - 2bc \cos \hat{A}) + b(c^2 + 2b^2c \cos \hat{A}) \Rightarrow a^2 = a^2 + b(c^2 + 2b^2c \cos \hat{A}) - c(b^2 + 2b^2c \cos \hat{A})$$

$$\frac{bc(c-b)}{-bc(b-c)} = \frac{2bc \cos \hat{A} (b-c)}{-bc(b-c)} \rightarrow \cos \hat{A} = \boxed{-\frac{1}{2}}$$



$$\frac{1}{2} \times (1 - \cos \theta) (1 + 1 \cos \theta) \times \frac{1}{2} = \frac{1}{2} (1 + \cos \theta - \cos \theta - \cos^2 \theta) = 1$$

$$- \cos^2 \theta + 1 \cos \theta - 2 = 0 \rightarrow \cos \theta = 1$$

$$a^2 = b^2 + c^2 - 2bc \cos 120^\circ \rightarrow a^2 = 2 + 1 - 2 \times 1 \times 1 \times \frac{\sqrt{3}}{2} \rightarrow \frac{a}{1} \overrightarrow{OO} \hat{z}$$

$$\boxed{a^2 = 1 - 1\sqrt{3}}$$