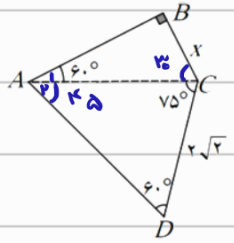


$$B + C = 120^\circ \rightarrow \hat{A} = 180^\circ - 120^\circ = 60^\circ$$

$$\Rightarrow \hat{B} = 45^\circ \Rightarrow \hat{C} = 180^\circ - 45^\circ - 75^\circ$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} \rightarrow \frac{4}{\sin 60^\circ} = \frac{3\sqrt{4}}{\sin B} \Rightarrow \sin B = \frac{\sqrt{3}}{4} = \frac{\sqrt{2}}{2}$$

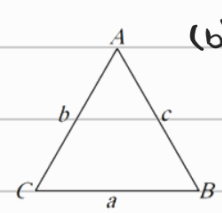
سوال ۱)



$$\frac{a}{\sin A} = \frac{d}{\sin D} \rightarrow \frac{4}{\sin 60^\circ} = \frac{d}{\sin 45^\circ} \Rightarrow AC = 4\sqrt{3}$$

$$\sin A = \frac{a}{AC} \rightarrow \frac{\sqrt{3}}{4} = \frac{a}{4\sqrt{3}} \rightarrow a = 3$$

سوال ۲)

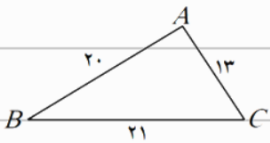


$$(b^2 - 2) \frac{\sin \hat{C}}{\sin \hat{B}} = c \rightarrow \frac{c}{\sin C} = \frac{b}{\sin B} \rightarrow \frac{\sin \hat{C}}{\sin \hat{B}} = \frac{c}{b}$$

$$(b^2 - 2) \times \frac{c}{b} = c \xrightarrow{c \neq 0} (b^2 - 2) \times \frac{1}{b} = 1 \rightarrow b^2 - b - 2 = 0$$

ب = -1 غیر قابل قبول
ب = 2 قابل قبول

سوال ۳)

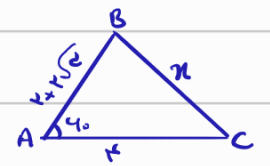


$$b^2 = a^2 + c^2 - 2ac \sin \hat{B} \rightarrow (13)^2 = (10)^2 + (21)^2 - 2(10)(21) \sin \hat{B}$$

$$\rightarrow 169 = 400 + 441 - 420 \times \cos \hat{B} \rightarrow \cos \hat{B} = \frac{4}{5}$$



سوال ۴)

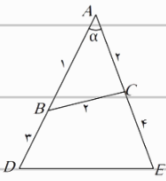


$$\text{الف) } x^2 = b^2 + c^2 - 2bc \cos \hat{A} \rightarrow x^2 = 14 + 14 + 1\sqrt{2} - 2 \times 2 \times (2 + \sqrt{2}) \times \frac{1}{2}$$

$$\rightarrow x^2 = 32 + 1\sqrt{2} - 1 - 1\sqrt{2} = 32 \rightarrow x = 2\sqrt{4}$$

$$\text{ب) } \frac{b}{\sin B} = \frac{a}{\sin A} \rightarrow \frac{4}{\sin B} = \frac{2\sqrt{4}}{\sin 40^\circ} \rightarrow \sin B = \frac{\sqrt{2}}{2} \rightarrow \hat{B} = 45^\circ \rightarrow \hat{C} = 180^\circ - (40^\circ + 45^\circ) = 95^\circ$$

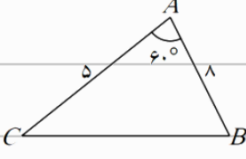
سوال ۵)



$$a^2 = b^2 + c^2 - 2bc \cos \hat{A} \rightarrow 4 = 4 + 1 - 2 \times 2 \times 1 \times \cos \hat{A} \rightarrow \cos \hat{A} = \frac{1}{2}$$

$$(DE)^2 = (AE)^2 + (AD)^2 - 2(AE)(AD) \times \cos \hat{A} \rightarrow (DE)^2 = 34 + 14 - 2 \times 4 \times 4 \times \frac{1}{2} \rightarrow (DE)^2 = 40 \rightarrow DE = 2\sqrt{10}$$

سوال ۶)



$$\text{الف) } a^2 = b^2 + c^2 - 2bc \cos \hat{A} \rightarrow a^2 = 25 + 44 - 2 \times 5 \times 11 \times \frac{1}{2} = 49 \Rightarrow a = 7$$

$$\text{ب) } S = \frac{1}{2} \times b \times c \times \sin \hat{A} \rightarrow \frac{1}{2} \times 5 \times 11 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$$

سوال ۷)

$$\frac{(b+c)^2 - a^2}{bc(\cos A + 1)} = \frac{b^2 + c^2 + 2bc - b^2 - c^2 + 2bc \cos A}{bc(\cos A + 1)} = \frac{2bc(\cos A + 1)}{bc(\cos A + 1)} = 2$$

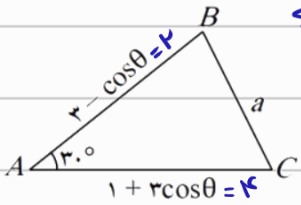
سوال ۸)

$$\frac{a^2 + b^2 - c^2}{a + b - c} = a^2 \rightarrow a^2 + b^2 - c^2 = a^2 + a^2(b - c) \rightarrow (b - c)(b^2 + c^2 + bc) = a^2(b - c)$$

سوال ۹

چون فرض سوال $b \neq c$ است پس $\rightarrow b^2 + c^2 + bc = a^2 \rightarrow b^2 + c^2 + bc = b^2 + c^2 - 2bc \cos \hat{A}$

$b - c \neq 0$ است پس می توان از دو طرف خط زد $\rightarrow \cos \hat{A} = \frac{bc}{-2bc} = -\frac{1}{2} \rightarrow \hat{A} = 120^\circ$



$$S_{\triangle ABC} = 2 = \frac{1}{2} \times b \times c \times \sin \hat{A} \rightarrow \frac{1}{2} \times (2 - \cos \theta) \times (1 + r \cos \theta) \times \frac{1}{2} = 2$$

سوال ۱۰

$$\rightarrow -r \cos^2 \theta + 1 \cos \theta + 1 = 4 \rightarrow -r \cos^2 \theta + 1 \cos \theta - 3 = 0 \rightarrow \cos \theta = 1 \vee \cos \theta = \frac{3}{r}$$

$$a^2 = b^2 + c^2 - 2bc \cos \hat{A} \rightarrow a^2 = 14 + 4 - 14 \frac{\sqrt{3}}{2} \rightarrow a^2 = 20 - 7\sqrt{3} \rightarrow a = \sqrt{20 - 7\sqrt{3}}$$