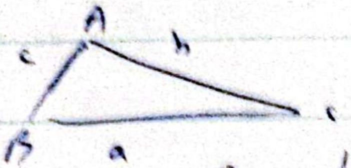
تکلیف (۱۳)

تکوینی — ریاضی دهم — حل مسئله‌ها

سؤال ۱

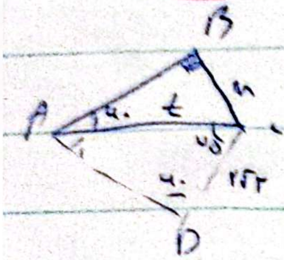


$\hat{A} = 180 - 120 = 60$ ، $BC = a = 20$

$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{20}{\sin 60} = \frac{10\sqrt{6}}{\sin B} \Rightarrow \frac{20}{\frac{\sqrt{3}}{2}} = \frac{10\sqrt{6}}{\sin B} \Rightarrow \frac{40}{\sqrt{3}} = \frac{10\sqrt{6}}{\sqrt{3}\sin B} \Rightarrow \frac{\sqrt{6}}{4} = \frac{\sqrt{6}}{4\sin B}$

$\sin B = \frac{\sqrt{2}}{2}$ → 135 ($60 + 135$ غ ق ق) / → $45 = B$ ، $\hat{C} = 180 - 105 = 75$

سؤال ۲

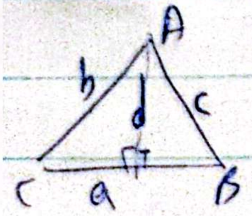


$A_1 = 180 - (60 + 75) = 45$ ، قانون سینوس‌ها → $\frac{3\sqrt{2}}{\sin 45} = \frac{t}{\sin 60}$

$\frac{3\sqrt{2}}{\frac{\sqrt{2}}{2}} = \frac{t}{\frac{\sqrt{3}}{2}} \Rightarrow t = 3\sqrt{3}$

$\overset{\triangle}{ABC} \Rightarrow \frac{t}{\sin B} = \frac{x}{\sin A} \Rightarrow \frac{3\sqrt{3}}{\sin 60} = \frac{x}{\frac{\sqrt{3}}{2}} \Rightarrow \boxed{3 = x}$

سؤال ۳

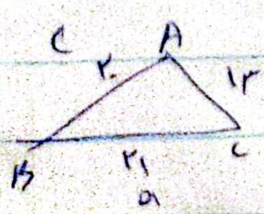


$(b^2 - 2)\frac{\sin C}{\sin B} = c \quad \div \sin C$

$\frac{b^2 - 2}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{b^2 - 2}{\frac{d}{c}} = \frac{c}{\frac{d}{b}} \Rightarrow \frac{b^2 - 2}{b} = \frac{c}{c}$

$b^2 - 2 = b \Rightarrow b^2 - b - 2 = 0 \quad (b - 2)(b + 1)$ → $\boxed{b = 2}$ / → $b = -1$ غ ق

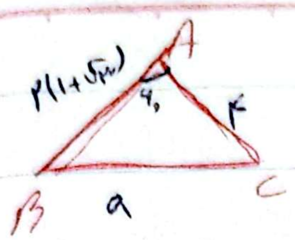
سؤال ۴



$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \Rightarrow \cos B = \frac{(21)^2 + (20)^2 - (13)^2}{2(20)(21)}$

$\cos B = \frac{441 + 400 - 169}{840} = \frac{672}{840} = \frac{8}{10} \Rightarrow \sin B = \sqrt{1 - \cos^2} = \boxed{\frac{6}{10}}$

سوال ۵



$a^2 = b^2 + c^2 - 2bc\cos 60$

$a^2 = 16 + 4(1+3+2\sqrt{3}) - 2\times 4\times 2(1+\sqrt{3})\times \frac{1}{2}$

$a^2 = 16+4+12+8\sqrt{3} - 8 - 8\sqrt{3} \Rightarrow a^2 = 24$

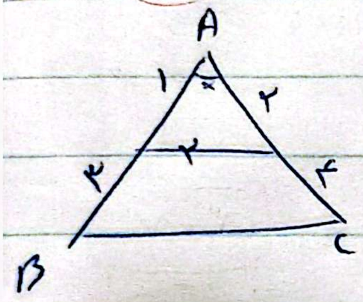
الف

$a = +\sqrt{24} \Rightarrow a = 2\sqrt{6}$

$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{2\sqrt{6}}{\sin 60} = \frac{4}{\sin B} \Rightarrow \frac{2\sqrt{6}}{\frac{\sqrt{3}}{2}} = \frac{4}{\sin B} \Rightarrow \sqrt{2} = \frac{1}{\sin B}$

$\sin B = \frac{\sqrt{2}}{2}$ → $B = 45$ → $C = 75$
→ ۱۳۵ غ ق ق

سوال ۶

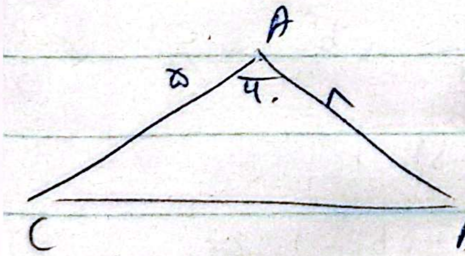


$\cos A = \frac{b^2+c^2-a^2}{2bc} \Rightarrow \cos\alpha = \frac{(2)^2+(1)^2-(2)^2}{2(2)(1)}$

$\cos\alpha = \frac{4+1-4}{4} \Rightarrow \cos\alpha = \frac{1}{4}$

$BC = \sqrt{c^2+b^2-2bc\cos A} \Rightarrow 16+36-2\times 4\times 6\times \frac{1}{4} \Rightarrow \sqrt{40}$

سوال ۷



الف) $BC = \sqrt{b^2+c^2-2bc\cos 60}$

$BC = \sqrt{64+25-2\times 8\times 5\times \frac{1}{2}} \Rightarrow \sqrt{49} = 7$

ب) $S = \frac{1}{2}ab\sin \Rightarrow \frac{1}{2}\times 5\times 8\times \frac{\sqrt{3}}{2} = 10\sqrt{3}$

سوال ۸

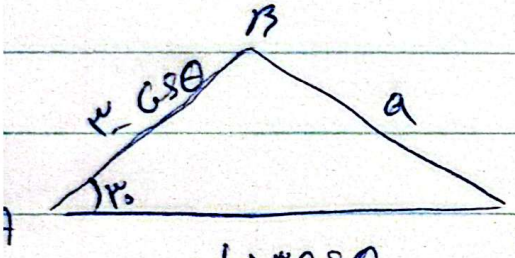
$\frac{(b+c)^2-a^2}{bc(\cos A+1)} \Rightarrow \frac{b^2+c^2+2b-a^2}{bc(\cos A+1)} \Rightarrow \frac{b^2+c^2-a^2+2bc}{bc(\cos A+1)}$

$$\frac{2bc\cos A + 2bc}{bc(\cos A+1)} = \frac{2bc(\cos A+1)}{bc(\cos A+1)} = 2$$

$$\frac{a^2+b^2-c^2}{a+b-c} = a^2 \Rightarrow a^2+b^2-c^2 = a^3+a^2b-a^2c$$

$$b^2-c^2 = a^2(b-c) \Rightarrow (b-c)(b^2+c^2+bc) = a^2(b-c)$$

$$a^2 = b^2+c^2+bc \rightarrow a^2 = b^2+c^2-2bc\cos A \rightarrow \cos A = -\frac{1}{2}$$



$$S = \frac{1}{2}ab\sin C$$

$$S = \frac{1}{2}\times(2-\cos\theta)(1+2\cos\theta)\times\frac{1}{2} = 2$$

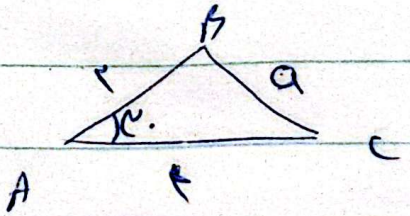
$$2-\cos\theta+4\cos\theta-2\cos^2\theta = 4$$

$$2\cos^2\theta-3\cos\theta+1 = 0$$

$$\cos^2\theta-3\cos\theta+2 = 0 \Rightarrow (\cos\theta-2)(\cos\theta-1) = 0$$

$$\rightarrow \cos\theta = \frac{2}{2} = 1 \quad \text{مرفوض}$$

$$\rightarrow \cos\theta = \frac{1}{2} \quad \text{مقبول}$$



$$a^2 = b^2+c^2-2bc\cos A \rightarrow a^2 = 4+16-2\times2\times4\times\left(-\frac{1}{2}\right) \rightarrow a^2 = 20-8\sqrt{3}$$