۱۹

مرسا نجاتی / تکلیف شماره‌ی ۱۲

①

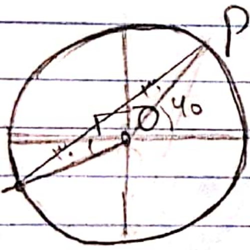
$$\frac{a\sin\frac{4\pi}{3}+b\cos\frac{7\pi}{6}}{a\sin\frac{11\pi}{6}+b\cos\frac{\pi}{3}}=\tan\frac{5\pi}{6} \qquad \frac{b}{a}\,?$$

$$\frac{a(-\frac{\sqrt{3}}{2})+b(-\frac{\sqrt{3}}{2})}{a(-\frac{1}{2})+b(\frac{1}{2})}=-\frac{\sqrt{3}}{3}\to\frac{-\frac{\sqrt{3}}{2}(a+b)}{-\frac{1}{2}(a-b)}=-\frac{\sqrt{3}}{3}\to$$

$$3a+3b=-a+b\to 4a=-2b\to a=\frac{-b}{2}$$

$$\frac{b}{a}=\frac{b}{\frac{-b}{2}}=-2$$

②



$$P(\tfrac{1}{2},y)=(\tfrac{1}{2},\tfrac{\sqrt{3}}{2})$$

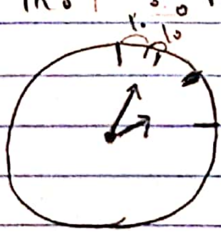
$$P'\ (-\tfrac{\sqrt{3}}{2},-\tfrac{1}{2})$$

$$PP'=\sqrt{(\tfrac{\sqrt{3}}{2}-(-\tfrac{1}{2}))^2+(\tfrac{1}{2}-(-\tfrac{\sqrt{3}}{2}))^2}=\sqrt{2+\sqrt{3}}$$

محیط $=1+1+\sqrt{2+\sqrt{3}}=2+\sqrt{2+\sqrt{3}}$

③

$$\frac{D}{180}=\frac{Rad}{\pi}\to\frac{D}{180}=\frac{\frac{\pi}{9}}{\pi}\to D=\frac{180}{9}=20^\circ$$



حرکت دقیقه شمار ۱۲ برابر حرکت ساعت شمار است $\Rightarrow 20\times 2=40$

۲:۴۰

(۴)

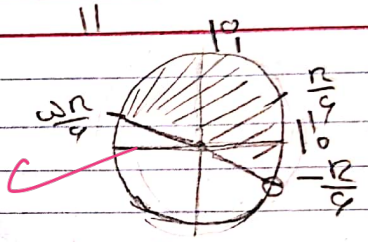
$$\alpha = \left|\frac{11M}{2} - 110\right| \rightarrow 20 = \left|\frac{11M}{2} - \overset{90}{\cancel{100}}(3)\right| \rightarrow$$

$$I: \frac{11M}{2} - 90 = 20 \rightarrow \frac{11M}{2} = 110 \rightarrow M = \frac{110 \times 2}{11} = 20 \rightarrow$$ برای بار دوم

$$II: \frac{11M}{2} - 90 = -20 \rightarrow \frac{11M}{2} = 70 \rightarrow M = \frac{70 \times 2}{11} \simeq 13$$

(۵)

$$-\frac{\pi}{6} < \alpha \leq \frac{3\pi}{4} \qquad \sin\alpha = \frac{m-3}{5}$$



$$-\frac{1}{2} < \sin\alpha \leq 1 \rightarrow$$

$$-\frac{1}{2} < \frac{m-3}{5} \leq 1 \xrightarrow{\times 5} -\frac{5}{2} < m-3 \leq 5 \xrightarrow{+3} \frac{1}{2} < m \leq 8$$

(۶)

$$\frac{\sin(100^\circ) + k\cos(225^\circ)}{\cos(140^\circ) + 2\sin(190^\circ)} = 3 \qquad \tan 45^\circ = 0/25$$

$$\frac{\sin\left(\frac{\pi}{2} + 1\omega\right) + k\cos\left(\frac{3\pi}{2} - 1\omega\right)}{\cos(\pi - 1\omega) + 2\sin(\pi + 1\omega)} = 3 \rightarrow$$

$$\frac{\cos 1\omega - k\sin 1\omega}{-\cos 1\omega - 2\sin 1\omega} = 3 \xrightarrow{\div \cos 1\omega} \frac{1 - k(0/25\omega)}{-1 - 2(0/25\omega)} = 3 \rightarrow$$

$$1 - k(0/25\omega) = -3 \rightarrow 3/5 = k(0/25\omega) \rightarrow k = \frac{3/5}{0/25\omega} = 22$$

(۷)

$\frac{1-\cos x}{1+\cos x}=2 \rightarrow$ انتهای کمان در ناحیه‌ی ۲ $\frac{\tan(\frac{\pi}{2}-x)-\cot(\pi-x)}{\cot(\frac{3\pi}{2}-x)+2\tan(2\pi-x)}$ (۲)

$1-\cos x=2+2\cos x \rightarrow 3\cos x=-1 \rightarrow \cos x=\frac{-1}{3}$ $\tan=\frac{\frac{2\sqrt{2}}{3}}{-\frac{1}{3}}=-2\sqrt{2}$

$\sin^2 x+\cos^2 x=1 \rightarrow \sin^2 x=1-\cos^2 x=1-\frac{1}{9}=\frac{8}{9} \rightarrow \sin x=\frac{2\sqrt{2}}{3}$

$\frac{\cot x+\cot x}{\tan x-2\tan x}=\frac{2\cot x}{-\tan x} \xrightarrow{\cot x=\frac{1}{\tan x}} \frac{2}{-\tan^2 x}=\frac{2}{-8}=\frac{-1}{4}$ ✓

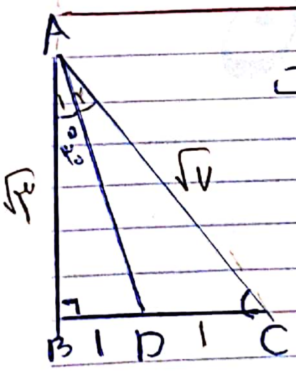
(۸)

$L=\theta r$ $L_A=L_B \rightarrow \frac{3\pi}{5}r_A=\frac{9\pi}{10}r_B \rightarrow \frac{r_A}{r_B}=\frac{\frac{9\pi}{10}}{\frac{3\pi}{5}}=\frac{3}{2}$

$\frac{D}{180}=\frac{R}{\pi} \rightarrow \frac{3\pi}{5}=R$ (D = 108) (۲)

$\frac{\text{مساحت } A}{\text{مساحت } B}=\frac{\pi(r_A)^2}{\pi(r_B)^2}=(\frac{3}{2})^2=\frac{9}{4}$ ✓

(۹)



مساحت $\overset{\triangle}{ABC}=\sqrt{3}$ $\sin 2\hat{C}=?$

$\sin 30^\circ=\frac{1}{2}=\frac{1}{AD} \rightarrow AD=2$

$\cos 30^\circ=\frac{\sqrt{3}}{2}=\frac{AB}{2} \rightarrow AB=\sqrt{3}$

$\frac{AB\times BC}{2}=\sqrt{3} \rightarrow \frac{\sqrt{3}\times BC}{2}=\sqrt{3} \rightarrow BC=2 \xrightarrow{BD=1} DC=1$

$AC=\sqrt{AB^2+BC^2}=\sqrt{3+4}=\sqrt{7}$

$\sin 2C=2\sin C\cos C=2\frac{\sqrt{3}}{\sqrt{7}}\times\frac{2}{\sqrt{7}}=\frac{4\sqrt{3}}{7}$ ✓

$AN = AM \rightarrow AM^2 = (AB^2) + (BM^2) = ۲^2 + ۳^2 = ۱۳ \rightarrow AM = \sqrt{۱۳}$

$S_{AON} = S_{ABM} = \frac{۲ \times ۳}{۲} = ۳$ ، $S_{MCN} = \frac{۱ \times ۱}{۲}$

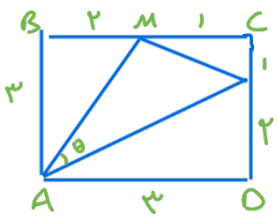
$S_{AON} + S_{ABM} + S_{MCN} + S_{AMN} = S_{ABCD} \rightarrow$

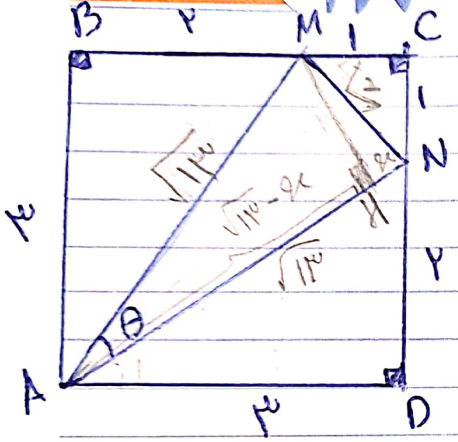
۳ ۳ $\frac{۱}{۲}$ ۹

$\frac{۱}{۲} \times \sqrt{۱۳} \times \sqrt{۱۳} \times \sin\theta = \frac{۹}{۲}$

$\sin\theta = \frac{۹}{۱۳}$

۲,۵





$\sin\theta = ?$

۱۰

$MA = \sqrt{۹+۴} = \sqrt{۱۳}$

۱

راه حل خوبی نیست چون جواب خیلی مطلوبی می شود !!

$MH^2 = \sqrt{۱۳}^2 - (\sqrt{۱۳}-x)^2 = ۱۳ - ((۱۳ + x^2 - ۲\sqrt{۱۳}x) =$

$MH^2 = x(۲\sqrt{۱۳} - x)$

$MH = \sqrt{۱۳ - (۱۳ - ۲x\sqrt{۱۳} + x^2)} = \sqrt{۲ - x^2} \rightarrow$

$MH^2 = ۲x\sqrt{۳} - x^2 = ۲ - x^2 \rightarrow x = \frac{\sqrt{۳}}{۳}$

$MH = \sqrt{۲ - \frac{۱}{۳}} = \sqrt{\frac{۷}{۳}}$

$\sin\theta = \frac{\sqrt{\frac{۷}{۳}}}{\sqrt{۱۳}} = \sqrt{\frac{۷}{۳۹}}$

$MH^2 = ۲ - x^2$

$۲ - x^2 = ۲\sqrt{۱۳}x - x^2 \rightarrow$ $x\sqrt{۱۳} = MH^2$