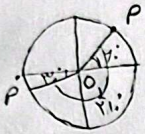
۱

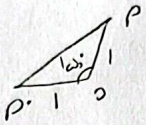
$$\frac{a\sin\frac{4\pi}{3}+b\cos\frac{7\pi}{6}}{a\sin\frac{11\pi}{6}+b\cos\frac{\pi}{3}}=\frac{a\left(-\frac{\sqrt{3}}{2}\right)+b\left(-\frac{\sqrt{3}}{2}\right)}{a\left(-\frac{1}{2}\right)+b\left(\frac{1}{2}\right)}=\frac{-\sqrt{3}(a+b)}{-\sqrt{3}a-\sqrt{3}b}=\tan\frac{5\pi}{6}=-\frac{\sqrt{3}}{3}\Rightarrow\frac{a+b}{b-a}=\frac{1}{3}\Rightarrow 3a+3b=b-a\Rightarrow 2b=-4a\Rightarrow\boxed{\frac{b}{a}=-2}$$

۲



$P\left(\frac{1}{2},y\right)=\frac{\pi}{3}$

$P'=\pi+\frac{\pi}{6}=\frac{5\pi}{6}\Rightarrow\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$



$PP'=\sqrt{OP^2+OP'^2-2\cos 150^\circ\cdot OP\cdot OP'}=\sqrt{2+\sqrt{3}}$

$\Rightarrow 2P=1+1+\sqrt{2+\sqrt{3}}=\boxed{2+\sqrt{2+\sqrt{3}}}$ محیط

۳

$$\frac{\frac{M}{9}}{\frac{2M}{12}}=\frac{M}{60}\Rightarrow M=40\Rightarrow\boxed{T=20.40}$$

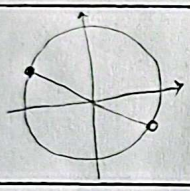
۴

$|30M-15M|=20^\circ=|90-15M|$

$90-15M=20\Rightarrow 70=15M\Rightarrow M=\frac{140}{11}$

$90-15M=-20\Rightarrow\boxed{M=20}$

۵



$-1<\sin x<1\Rightarrow -1<\frac{m-3}{5}<1\Rightarrow -5<m-3<5\Rightarrow -2<m<8\rightarrow$ ۸ عدد صحیح

۶

$$\frac{\sin(150^\circ)+K\cos(270^\circ)}{\cos(165^\circ)+2\sin(315^\circ)}=\frac{\sin(90^\circ+15^\circ)+K\cos(270^\circ-15^\circ)}{\cos(180^\circ-15^\circ)+2\sin(180^\circ+15^\circ)}=\frac{\cos 15^\circ-K\sin 15^\circ}{-\cos 15^\circ-2\sin 15^\circ}=3\xrightarrow[\text{صورت و مخرج}]{\div\cos 15^\circ}\frac{1-K\tan 15^\circ}{-1-2\tan 15^\circ}=3$$

$\Rightarrow 1-K\tan 15^\circ=-3-6\tan 15^\circ\Rightarrow(6-K)\underbrace{\tan 15^\circ}_{0/25}=-4\Rightarrow\boxed{K=22}$

۷

$$\frac{\tan\left(\frac{\pi}{2}-x\right)-\cot(\pi-x)}{\cot\left(\frac{3\pi}{2}-x\right)+2\tan(2\pi-x)}=\frac{\cot x+\cot x}{-\tan x-2\tan x}=\frac{2\cot x}{-3\tan x}=\frac{2\left(\frac{1}{\tan x}\right)}{-3\tan x}=\frac{2}{-3\tan^2 x}$$

$\frac{1-\cos x}{1+\cos x}=\tan^2\frac{x}{2}=2\Rightarrow\tan x=\pm\sqrt{2}$

tan در ناحیه دوم منفی است

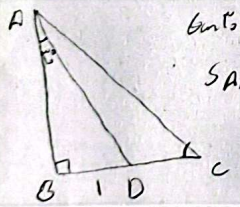
$\tan x=\frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}=\frac{-2\sqrt{2}}{-1}=2\sqrt{2}\Rightarrow\frac{2}{-3\tan^2 x}=\frac{2}{-3(2\sqrt{2})^2}=\boxed{\frac{-1}{12}}$

۸

$\frac{150\pi}{180}=\frac{R}{r}\Rightarrow R=\frac{3r}{5}$

$l=\frac{3\pi}{5}\times r_A=\frac{9\pi}{5}\times r_B\Rightarrow\frac{r_A}{r_B}=3\Rightarrow\frac{S_A}{S_B}=\left(\frac{r_A}{r_B}\right)^2=\boxed{9}$

۹

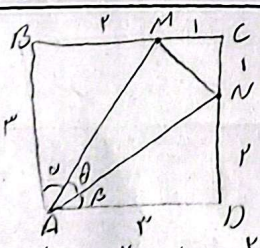


$\tan 30^\circ=\frac{1}{\sqrt{3}}\Rightarrow AB=\sqrt{3}$

$S_{ABC}=AB\times BC\times\frac{1}{2}=\sqrt{3}\Rightarrow BC=2\Rightarrow\tan C=\frac{AB}{BC}=\frac{\sqrt{3}}{2}$

$\sin 2C=\frac{2\tan C}{1+\tan^2 C}=\frac{\sqrt{3}}{\frac{7}{4}}=\boxed{\frac{4\sqrt{3}}{7}}$

۱۰



$\tan(90^\circ-(\alpha+\beta))=\tan\theta=\cot(\alpha+\beta)$

$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta}=\frac{\frac{6}{3}}{\frac{5}{9}}=\frac{12}{5}\Rightarrow\cot(\alpha+\beta)=\frac{5}{12}=\tan\theta\Rightarrow\cot\theta=\frac{12}{5}$

$\Rightarrow 1+\cot^2\theta=\frac{1}{\cos^2\theta}=1+\frac{144}{25}\Rightarrow\boxed{\cos\theta=\frac{5}{13}}$

$\tan\alpha=\frac{2}{3}$  $\tan\beta=\frac{2}{3}$

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