1 -

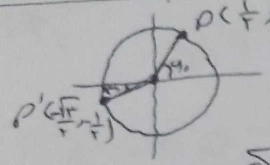
$$\frac{a(-\frac{\sqrt{3}}{2}) + b(-\frac{\sqrt{3}}{2})}{a(-\frac{1}{2}) + b(\frac{1}{2})} = -\frac{\sqrt{3}}{3} \Rightarrow \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} \cdot \frac{a+b}{b-a} = -\frac{\sqrt{3}}{3} \Rightarrow -\sqrt{3}\frac{(a+b)}{(b-a)} = -\frac{\sqrt{3}}{3}$$

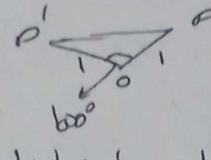
$$\frac{a+b}{b-a} = \frac{1}{3} \Rightarrow 3a+3b = b-a \quad 4a = -2b \Rightarrow 2a = -b$$

$$-2a = b$$

$$\frac{b}{a} = \frac{-2a}{a} = -2$$

2 - مساحت مثلث برابر است.





$$S_{OPP'} = \frac{1}{2} OP \times OP' \times \sin 150°$$

$$S_{OPP'} = \frac{1}{2} \times 1 \times 1 \times \frac{1}{2} = \frac{1}{4}$$

3

$$\frac{\frac{\pi}{9}}{\frac{\pi}{1}} = \frac{D}{180} \Rightarrow D = 20°$$

$$\frac{\frac{\pi}{9}}{\frac{\pi}{6}} = \frac{x}{60°} \Rightarrow \frac{2}{3} = \frac{x}{60} \Rightarrow x = 40°$$

۶۰ دقیقه

$$2° 40'$$

4 -

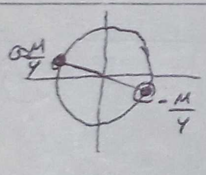
زاویه بین ساعت شمار و دقیقه شمار $= |30H - 5.5m|$

$$20 = |30 \times 3 - 5.5m|$$

$$\begin{cases} 90 - 5.5m = 20 \Rightarrow m = 70 \times \frac{2}{11} \\ 90 - 5.5m = -20 \Rightarrow m = 110 \times \frac{2}{11} = 20 \end{cases}$$

پس از ۲۰ دقیقه این اتفاق می‌افتد

5



$\min = -\frac{1}{2}$

$\max = 1$

$$-\frac{1}{2} < \sin x \le 1 \Rightarrow -\frac{1}{2} < \frac{m-3}{5} \le 1$$

$$\Rightarrow \frac{1}{2} < m \le 8$$

6

$$\frac{\sin 15°}{\cos 15°} = \frac{25}{100} = \frac{1}{4} \Rightarrow \begin{matrix} \sin 15° = 1x \\ \cos 15 = 4x \end{matrix}$$

$$\frac{\sin(90+15) + k\cos(270-15)}{\cos(180-15) + 2\sin(180+15)}$$

$$3 = \frac{\cos 15 + (-k)\sin 15}{-\cos 15 - 2\sin 15} \Rightarrow \frac{4x - kx}{-4x - 2x} \Rightarrow \frac{x(4-k)}{-6x} = +3 \Rightarrow 4 - k = -18$$

$$k = 22$$

7

$$\frac{1-\cos x}{1+\cos x} = 2 \Rightarrow 2 + 2\cos x = 1 - \cos x \Rightarrow 3\cos x = -1 \quad \cos x = -\frac{1}{3}$$

$$\tan(\frac{\pi}{2} - x) = \cot x \qquad \cot(\frac{3\pi}{2} - x) = \tan x$$

$$\cot(\pi - x) = -\cot x \qquad 2\tan(2\pi - x) = -2\tan x$$

(۷)

$$\frac{\cot x - (-\cot x)}{\tan x - 2\tan x} = \frac{2\cot x}{-\tan x} = \frac{\frac{2\cos x}{\sin x}}{\frac{\sin x}{-\cos x}} = \frac{-2\cos^2 x}{\sin^2 x}$$

$\cos x = \frac{-1}{3}$ $\quad \cos^2 x + \sin^2 x = 1$

$\frac{1}{9} + \sin^2 x = 1 \Rightarrow \sin^2 x = \frac{8}{9}$

$$\frac{-2 \times \frac{1}{9}}{\frac{8}{9}} = -\frac{1}{4}$$

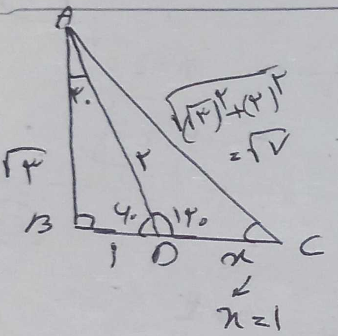
(۸)

$L = r\theta \Rightarrow \frac{6\pi}{10} \times r_A = r_B \times \frac{9\pi}{10}$ $\quad 2r_A = 3r_B$

$\frac{108}{180} = \frac{R}{\pi} \Rightarrow R = \frac{6\pi}{10}$

$\frac{S_A}{S_B} = \left(\frac{r_A}{r_B}\right)^2 \Rightarrow \frac{S_A}{S_B} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

(۹)



ضلع روبه‌رو به زاویه ۳۰° نصف وتر است و ضلع روبه‌رو زاویه ۶۰ درجه $\frac{\sqrt{3}}{2}$ وتر است

در ABD: $AB = \sqrt{3}$ ، $AD = 2$

$S_{ABC} = \frac{1}{2}(AB)(BC) \Rightarrow S_{ABC} = \frac{1}{2} \times \sqrt{3} \times (1+x) = \sqrt{3}$

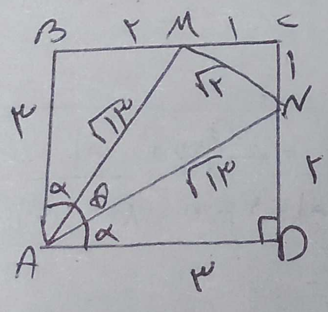
$\frac{1+x}{2} = 1 \Rightarrow 1+x = 2 \Rightarrow x = 1$

$AC = \sqrt{7}$

$\sin 2C = 2\sin C \cos C$
$\sin C = \frac{\sqrt{3}}{\sqrt{7}}, \cos C = \frac{2}{\sqrt{7}}$ $\Rightarrow 2 \times \frac{\sqrt{3}}{\sqrt{7}} \times \frac{2}{\sqrt{7}} = \frac{4\sqrt{3}}{7}$

(۱۰)

$\sin\theta = ?$



$(MC)^2 + (NC)^2 = (MN)^2 \Rightarrow 1 + 1 = 2$

$MN = \sqrt{2}$

$(ND)^2 + (AD)^2 = (AN)^2 \Rightarrow AN = \sqrt{13}$

$4 + 9 = 13$

$(BM)^2 + (AB)^2 = (AM)^2$

$AM = \sqrt{13}$

مثلث $A\hat{B}M$ و $A\hat{D}N$ باهم همنهشت هستند.

$A_1 = A_2 = \alpha$

2α و θ متمم هستند

$\sin\theta = \cos 2\alpha$

$\cos\alpha = \frac{3}{\sqrt{13}}$ $\quad \sin\alpha = \frac{2}{\sqrt{13}}$

$\sin\theta = (\cos^2\alpha - \sin^2\alpha)$

$\sin\theta = \left(\frac{3}{\sqrt{13}}\right)^2 - \left(\frac{2}{\sqrt{13}}\right)^2 \Rightarrow \sin\theta = \frac{9}{13} - \frac{4}{13} = \frac{5}{13}$