۱۸/۵

[illegible] قنبری — تکلیف ۱۲ — ۱۴۰۴/۸/۵

۱ـ (۲)

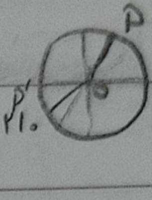
$$\frac{a\times\frac{-\sqrt{۳}}{۲}+b\,\frac{-\sqrt{۳}}{۲}}{a\times\frac{-۱}{۲}+b\,\frac{۱}{۲}}=\frac{-\sqrt{۳}}{۳}\Rightarrow\left(\frac{-\sqrt{۳}a}{۲}+\frac{-\sqrt{۳}b}{۲}\right)۳=\frac{a\sqrt{۳}}{۴}-\frac{b\sqrt{۳}}{۴}$$

$$\Rightarrow\frac{-۳\sqrt{۳}a}{۴}+\frac{-۳\sqrt{۳}b}{۴}=\frac{a\sqrt{۳}}{۴}-\frac{b\sqrt{۳}}{۴}$$

$$\Rightarrow\frac{-۳\sqrt{۳}a}{۴}-\frac{a\sqrt{۳}}{۴}=\frac{-b\sqrt{۳}}{۴}+\frac{۳\sqrt{۳}b}{۴}\Rightarrow -۲a=b\Rightarrow\frac{b}{a}=-۲$$

۲ـ (۱/۵)

$P(\frac{۱}{۲},\frac{\sqrt{۳}}{۲})=۹۰^\circ \xrightarrow{۲۱۰^\circ \text{ در خلاف}} P'=۲۱۰^\circ$



محیط $OPP' = \overline{PO}+\overline{P'O}+\overline{PP'}=۱+۱+۱=۳$

$\overline{PP'}=۲R\sin\alpha=۲\times۱\times\sin۱۵۰^\circ=۱$

$\sin۱۵۰^\circ=-\frac{\sqrt{۳}}{۲}\rightarrow PP'=\sqrt{۲+\sqrt{۳}}$ محیط $=۲+\sqrt{۲+\sqrt{۳}}$

۳ـ (۲)

$\frac{\pi}{۴}\,|\,۱h \quad ?h=\frac{۴}{۳}h=۴۰\,min \rightarrow ۲:۴۰$

$\frac{۱۳}{۹}\,|\,?h$

۴ـ (۲)

$\theta=\left|\frac{۱۱M}{۲}-۳۰H\right|\Rightarrow ۲۰=\left|\frac{۱۱M}{۲}-۹۰\right|\Rightarrow \begin{cases}۲۰=\frac{۱۱M}{۲}-۹۰\Rightarrow M=۲۰\\ -۲۰=\frac{۱۱M}{۲}-۹۰\Rightarrow M=\frac{۱۴۰}{۱۱}\simeq ۱۲/۷\end{cases}$

۵ـ (۱)

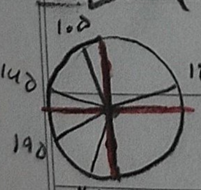
$-\frac{۱}{۲}<\sin x\le ۱\rightarrow -۰/۵<\frac{m-۲}{۵}\le۱\rightarrow -۲/۵\le m-۲\le ۵$

$-۱<\sin x\le\frac{۱}{۲}\rightarrow -۱<\frac{m-۴}{۵}\le\frac{۱}{۲}$

$۰/۵\le m\le ۸$

$\Rightarrow -۲<m\le ۵/۵ \rightarrow$ ۸ عدد صحیح

۶ـ (۲)



$$\frac{+\cos۱۵^\circ-k\sin۱۵^\circ}{-\cos۱۵^\circ-۲\sin۱۵^\circ}=۳ \xrightarrow[\sin=۵,\ \cos=۴۵]{\tan=\frac{۱}{۴}} \frac{۴-k}{-۴-۲}=-۳$$

$۴-k=-۱۸\Rightarrow k=۲۲$

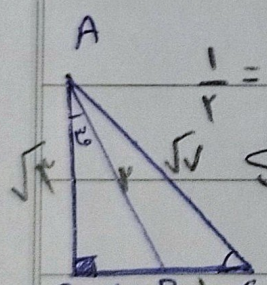
$$\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} = \tan^2\frac{x}{2} = 2 \Rightarrow \tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} = \frac{-2\sqrt{2}}{-1} \quad -7$$

$$\Rightarrow \frac{+\cot x + \cot x}{+\tan x - 2\tan x} = \frac{2\cot x}{-\tan x} = -2\cot^2 x = -2 \times \frac{1}{\tan^2 x}$$

$$\Rightarrow \tan^2 x = 8 \rightsquigarrow -2\cot^2 x = -\frac{2}{8} = -\frac{1}{4}$$

$$L = \theta r \Rightarrow \frac{108\pi}{180} \times r_A = \frac{9\pi}{10} \times r_B \Rightarrow \frac{r_A}{r_B} = \frac{\frac{9\pi}{10}}{\frac{3\pi}{5}} \quad -8$$

$$\Rightarrow \frac{r_A}{r_B} = \frac{3}{2} \xrightarrow{\text{②}} \frac{S_A}{S_B} = \frac{9}{4}$$

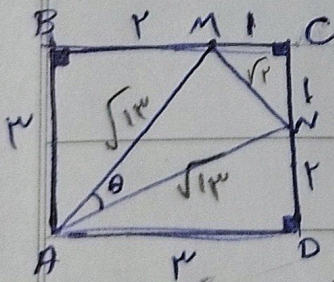


$$\frac{1}{2} = \sin 30 = \frac{BD}{AD} \Rightarrow AD = 2,\ AB = \sqrt{4-1} = \sqrt{3} \quad -9$$

$$S_{ABC} = \frac{AB \times BC}{2} = \sqrt{2} \Rightarrow BC = 2 \Rightarrow DC = 1$$

$$\sin\hat{C} = \frac{\sqrt{2}}{\sqrt{7}},\ \cos\hat{C} = \frac{2}{\sqrt{7}} \Rightarrow \sin 2\hat{C} = 2\sin\hat{C}\cos\hat{C}$$

$$= 2 \times \frac{\sqrt{2}}{\sqrt{7}} \times \frac{2}{\sqrt{7}} = \frac{4\sqrt{2}}{7}$$



$$AM = AN,\ BM^2 + BA^2 = AM^2 = AN^2 \quad -10$$

$$\Rightarrow \sqrt{4+9} = \sqrt{13} = AM = AN$$

$$\Rightarrow \text{قانون کسینوس} \quad 4 = MN^2 = AM^2 + AN^2 - 2AM.AN.\cos\theta$$

$$\Rightarrow 4 = 13 + 13 - 2 \times 13 \times \cos\theta \Rightarrow 26\cos\theta = 26 - 4 \Rightarrow \cos\theta = \frac{12}{13}$$

$$\Rightarrow \sin\theta = \frac{5}{13}$$