

$$f(x) = x + [x]$$

مثال (۱) $f(x) = x + [x] \Rightarrow y = x + [x] \xrightarrow[\text{برای}]{\text{از طرف}} [y] = [x + [x]] \Rightarrow [x] = \frac{[y]}{2}$

$$\Rightarrow f^{-1} = x + \frac{[y]}{2} \Rightarrow y = x - \frac{[x]}{2} \Rightarrow f^{-1}(x) = x - \frac{[x]}{2}$$

$$x - \frac{[x]}{2} = x + [x] \Rightarrow r[x] = -[x] \Rightarrow \begin{matrix} \nearrow r[x] = 0 \\ \rightarrow [x] = 0 \end{matrix}$$

$$\therefore f^{-1} \circ f^{-1}(x) = x - \frac{r}{2} \Rightarrow x = \frac{r}{2} \Rightarrow x \in \mathbb{Z}$$

$$y = \frac{ax + r}{fx + b}$$

$$\text{میان افراز} = \frac{a}{c} = \frac{a}{f}$$

$$\text{مخرج و میان افراز} = fx = -b \Rightarrow x = -\frac{b}{f}$$

$$f \circ f(\omega - \sqrt{4}) = ?$$

$$\Rightarrow y = \frac{-bx + r}{fx + b}, -b + b = 0 \Rightarrow f(x) = f^{-1}(x)$$

$$\Rightarrow f \circ f(\omega - \sqrt{4}) = \omega - \sqrt{4}$$

مثال (۲)

$$f(x) = \frac{mx + r}{x + m + r}$$

مثال (۳)

$$\frac{1}{\alpha} + \frac{1}{\beta} = ?$$

$$\frac{mx + r}{x + m + r} = x \Rightarrow x^2 + mx + rx = mx + r$$

$$\Rightarrow x^2 + rx - r = 0 \Rightarrow \text{جمع ریشه ها} \quad S = -\frac{b}{a} = -r$$

$$\text{ضرب ریشه ها} \quad \frac{c}{a} = -r$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-r}{-r} = 1$$

سوال ۴

$$y = \sqrt{(rx+d)^2} - \sqrt{(ax-r)^2} \quad f^{-1}(x) = ?$$

$$\left\{ \begin{array}{ll} -rx+v \rightarrow \text{برای } x > \frac{r}{a} \\ \sqrt{x+r} & -\frac{r}{a} \leq x \leq \frac{r}{a} \\ rx-v & x < -\frac{a}{r} \end{array} \right.$$

$$\underbrace{|rx+d|}_{x = -\frac{d}{r}} - \underbrace{|ax-r|}_{x = \frac{r}{a}}$$

$$y = -rx+v \rightarrow -ry = x-v \Rightarrow y = \frac{-x+v}{r}$$

$$g(x) = rf(rx) - 1 \xrightarrow{g^{-1}(x)} y = rf(rx) - 1 \Rightarrow x + \frac{1}{r} = f(rx)$$

سوال ۵

$$g^{-1}(x) = a f^{-1}\left(\frac{x+b}{c}\right) + d \quad f^{-1}\left(\frac{x+1}{r}\right) = ry \Rightarrow y = \frac{f^{-1}\left(\frac{x+1}{r}\right)}{r}$$

$$\frac{ac+d}{rb} = ?$$

$$\Rightarrow \begin{matrix} c=r \\ a=\frac{1}{r} \\ d=0 \\ b=1 \end{matrix} \Rightarrow \frac{ac+d}{rb} = \frac{1}{r^2}$$

$$\frac{\frac{1}{r} \times r + 0}{r(1)} = \frac{\frac{r}{r}}{r} = \frac{r}{r^2} = \frac{1}{r}$$

$$f(x) = x + r\sqrt{x} \Rightarrow y = x + r\sqrt{x} \Rightarrow y = x + r\sqrt{x} + 1 - 1 \Rightarrow$$

سوال ۶

$$y = (\sqrt{x}+1)^2 - 1 \Rightarrow y+1 = (\sqrt{x}+1)^2 \Rightarrow \sqrt{y+1} = \sqrt{x}+1 \Rightarrow$$

$$\sqrt{y+1} - 1 = \sqrt{x} \Rightarrow x = (\sqrt{y+1} - 1)^2 \Rightarrow y = (\sqrt{x+1} - 1)^2$$

$$f(x) \text{ بر } [1, \infty) = f^{-1}(x) \text{ بر } [1, \infty) \Rightarrow f \text{ بر } [1, \infty) = (\cdot, +\infty) \Rightarrow f^{-1} \text{ بر } [1, \infty) = [-1, +\infty)$$

سوال ۷) $y = x - 1 + r^x \Rightarrow$ $x = x - 1 + r^x \Rightarrow$ $r^x = 1 \Rightarrow x = 0$

$x - 1 \rightarrow$ صورتی است
 $r^x \rightarrow$ صورتی است
 $\left. \begin{array}{l} x - 1 \\ r^x \end{array} \right\} \Rightarrow$ در اینجا خط $y = x$ هم را قطع می کنند $\Rightarrow$ $\left[\begin{array}{c} 0 \\ 0 \end{array} \right]$ نقطه

سوال ۸) $f(x) = x^3 + 4x$ $y = \sqrt{f^{-1}(x) - x}$

$f^{-1}(x) - x \geq 0$ $y = x^3 + 4x$

$x^3 \rightarrow$ صورتی است
 $4x \rightarrow$ صورتی است
 $\left. \begin{array}{l} x^3 \\ 4x \end{array} \right\} \Rightarrow x^3 + 4x = x \Rightarrow x^3 + 3x = 0 \Rightarrow x(x^2 + 3) = 0 \Rightarrow x = 0$
 در اینجا $x = 0$ است

$f(x) = -x + \sqrt{x+5}$ $x \geq 3$

سوال ۹)

$y = x - 3$

$y = -x + \sqrt{x+5} \Rightarrow y + x = \sqrt{x+5} \Rightarrow (y+x)^2 = x+5 \Rightarrow (y+x)^2 = y+8$

$\Rightarrow x^2 + y^2 + 2xy = y + 8 \Rightarrow y^2 + x^2 + 2xy - y - 8 = 0 \Rightarrow y^2 + (2x-1)y + (x^2-8) = 0$

$\Rightarrow y = \frac{-(2x-1) \pm \sqrt{\Delta}}{2} \Rightarrow y = \frac{-2x+1 \pm \sqrt{1-4x}}{2}$

$\Rightarrow y = \frac{-2(x+4)+1 \pm \sqrt{1-4(x+4)}}{2}$

$y = \frac{-2x-7 \pm \sqrt{1-4x}}{2} \Rightarrow x = 0$

سؤال ١٦

$$\sqrt{x} + \sqrt{y} = r \Rightarrow f^{-1}(x) = \sqrt{y} + \sqrt{x} = r \Rightarrow f^{-1}(x) = f(x)$$

$$\Rightarrow Df = [0, r] \text{ , } Rf = [0, r] \Rightarrow Df^{-1} = Rf = \boxed{[0, r]}$$