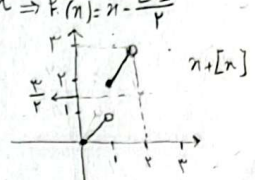
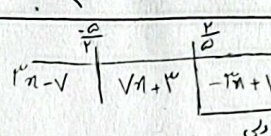


|    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                          |
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| ۱  | $f(n) = n + [n] \Rightarrow [n] = [n + [n]] = 2[n] \Rightarrow [n] = \frac{[n]}{2}$ <p>در این معادله، <math>[n]</math> یک عدد صحیح است و <math>\frac{[n]}{2}</math> نیز باید یک عدد صحیح باشد. بنابراین <math>[n]</math> باید زوج باشد. اگر <math>[n] = 2k</math>، داریم <math>2k = \frac{2k}{2} \Rightarrow k = k</math>. این معادله برای هر <math>k</math> برقرار است. اما اگر <math>[n] = 1</math>، داریم <math>1 = \frac{1}{2}</math> که نادرست است. بنابراین <math>[n] \neq 1</math>.<br/> <math>f^{-1}(n) = f(n) \Rightarrow n + [n] = n + \frac{[n]}{2} \Rightarrow \frac{[n]}{2} = 0 \Rightarrow [n] = 0 \Rightarrow n = 0</math><br/>         ب) <math>f \circ f^{-1}(n) = n \mid 0 = f \circ f^{-1}(n) = f(0) \Rightarrow n = 2 \cdot 0 = 0 \Rightarrow n = 0</math> </p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                       |
| ۲  | $y = \frac{ax+3}{x+b}$ <p>برای یافتن <math>f^{-1}</math>، داریم <math>y = \frac{ax+3}{x+b} \Rightarrow y(x+b) = ax+3 \Rightarrow yx + by = ax+3 \Rightarrow yx - ax = 3 - by \Rightarrow x(y-a) = 3-by \Rightarrow x = \frac{3-by}{y-a}</math><br/> <math>f^{-1}(n) = \frac{-bn+3}{n-a}</math><br/> <math>f^{-1}(n) = f(n) \Rightarrow \frac{-bn+3}{n-a} = \frac{an+3}{n+b} \Rightarrow (-bn+3)(n+b) = (an+3)(n-a) \Rightarrow -bn^2 - b^2n + 3n + 3b = an^2 - a^2n + 3n - 3a \Rightarrow -bn^2 - b^2n + 3n + 3b = an^2 - a^2n + 3n - 3a \Rightarrow -bn^2 - b^2n + 3b = an^2 - a^2n - 3a \Rightarrow (-b-a)n^2 + (-b^2+a^2)n + (3b+3a) = 0</math><br/>         این معادله برای هر <math>n</math> برقرار است. بنابراین ضرایب آن صفر باشند.<br/> <math>-b-a = 0 \Rightarrow a = -b</math><br/> <math>-b^2+a^2 = 0 \Rightarrow -b^2+(-b)^2 = 0 \Rightarrow -b^2+b^2 = 0 \Rightarrow 0 = 0</math><br/> <math>3b+3a = 0 \Rightarrow 3b+3(-b) = 0 \Rightarrow 0 = 0</math><br/>         بنابراین <math>a = -b</math> و <math>f^{-1}(n) = \frac{-bn+3}{n-a} = \frac{-bn+3}{n+b}</math><br/> <math>f \circ f^{-1}(n) = f\left(\frac{-bn+3}{n+b}\right) = \frac{a\left(\frac{-bn+3}{n+b}\right)+3}{\frac{-bn+3}{n+b}+b} = \frac{\frac{-abn+3a}{n+b}+3}{\frac{-bn+3+bn+3b}{n+b}} = \frac{\frac{-abn+3a+3n+3b}{n+b}}{\frac{3+3b}{n+b}} = \frac{-abn+3a+3n+3b}{3+3b} = \frac{-abn+3a+3n+3b}{3(1+b)}</math><br/>         اگر <math>a = -b</math>، داریم <math>f \circ f^{-1}(n) = \frac{-(-b)n+3(-b)+3n+3b}{3(1+b)} = \frac{bn-3b+3n+3b}{3(1+b)} = \frac{bn+3n}{3(1+b)} = \frac{n(b+3)}{3(1+b)}</math><br/>         برای اینکه <math>f \circ f^{-1}(n) = n</math>، باید <math>\frac{n(b+3)}{3(1+b)} = n \Rightarrow \frac{b+3}{3(1+b)} = 1 \Rightarrow b+3 = 3(1+b) \Rightarrow b+3 = 3+3b \Rightarrow b = 0</math><br/>         اگر <math>b = 0</math>، داریم <math>f(n) = \frac{an+3}{n} = a + \frac{3}{n}</math><br/> <math>f^{-1}(n) = \frac{-bn+3}{n-a} = \frac{3}{n-a}</math><br/> <math>f \circ f^{-1}(n) = f\left(\frac{3}{n-a}\right) = a + \frac{3}{\frac{3}{n-a}} = a + (n-a) = n</math><br/>         بنابراین <math>f^{-1}(n) = \frac{3}{n-a}</math> و <math>f \circ f^{-1}(n) = n</math> </p> | $D_f = \mathbb{R} - \{-\frac{3}{a}\}$<br>$R_f = \mathbb{R} - \{0\}$<br>$D_{f^{-1}} = \mathbb{R} - \{0\}$<br>$R_{f^{-1}} = \mathbb{R} - \{-\frac{3}{a}\}$ |
| ۳  | $f(n) = \frac{mn+3}{n+m+3} = n \Rightarrow mn+3 = n^2+m+3n \Rightarrow n^2+m+3n-mn-3 = 0 \Rightarrow n^2+(3-m)n-m+3 = 0$<br>$\Delta = (3-m)^2 - 4(1)(-m+3) = 9-6m+m^2+4m-12 = m^2-2m-3 = (m-3)(m+1)$<br>$m = 3$ یا $m = -1$<br>اگر $m = 3$ ، داریم $f(n) = \frac{3n+3}{n+3+3} = \frac{3(n+1)}{n+6}$<br>$f^{-1}(n) = \frac{3n+3}{n+6} = n \Rightarrow 3n+3 = n^2+6n \Rightarrow n^2+3n-3 = 0$<br>$\Delta = 9+12 = 21$<br>$n = \frac{-3 \pm \sqrt{21}}{2}$<br>اگر $m = -1$ ، داریم $f(n) = \frac{-n+3}{n-1+3} = \frac{-n+3}{n+2}$<br>$f^{-1}(n) = \frac{-n+3}{n+2} = n \Rightarrow -n+3 = n^2+2n \Rightarrow n^2+3n-3 = 0$<br>$\Delta = 9+12 = 21$<br>$n = \frac{-3 \pm \sqrt{21}}{2}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | $\frac{1}{a} + \frac{1}{b} = \frac{b+d}{ab} = \frac{-3}{-3} = 1$                                                                                         |
| ۴  | $f(n) = \sqrt{(n+5)^2} - \sqrt{(n-2)^2} =  n+5  -  n-2 $<br>$f^{-1}(n) = -\frac{1}{2}n + \frac{7}{2}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                        |
| ۵  | $g(n) = 2f(n) - 1 \Rightarrow g(n) = 2\left(\frac{n+1}{n}\right) - 1 = \frac{2n+2}{n} - 1 = \frac{2n+2-n}{n} = \frac{n+2}{n}$<br>$g^{-1}(n) = \frac{n}{n-2}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $a = \frac{1}{2}, b = 1, c = 2, d = 0 \Rightarrow \frac{ac+d}{2b} = \frac{1 \cdot 2 + 0}{2 \cdot 1} = 1$                                                 |
| ۶  | $f(n) = n + 2\sqrt{n} = (\sqrt{n}+1)^2 - 1$<br>$\Rightarrow \sqrt{n} = \sqrt{y+1} - 1 \Rightarrow n = (\sqrt{y+1} - 1)^2 = y + 2\sqrt{y+1} - 1$<br>$f^{-1}(n) = n + 2\sqrt{n+1}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | $D_f = [0, +\infty)$                                                                                                                                     |
| ۷  | $y = n - 1 + 2^n$<br>$f(n) = n - 1 + 2^n$<br>$f^{-1}(n) = n - 1 + 2^n$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | $n = 0$                                                                                                                                                  |
| ۸  | $y = \sqrt{f^{-1}(n)} - n \Rightarrow f^{-1}(n) = (y+n)^2$<br>$f(n) = (n)^2$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $\{0\}$                                                                                                                                                  |
| ۹  | $f(x) = \frac{x-2}{x+2}$<br>$f^{-1}(x) = \frac{x+2}{x-2}$<br>$f \circ f^{-1}(x) = f\left(\frac{x+2}{x-2}\right) = \frac{\frac{x+2}{x-2} - 2}{\frac{x+2}{x-2} + 2} = \frac{\frac{x+2-2x+4}{x-2}}{\frac{x+2+2x-4}{x-2}} = \frac{-x+6}{3x-2}$<br>$f^{-1} \circ f(x) = f^{-1}\left(\frac{x-2}{x+2}\right) = \frac{\frac{x-2}{x+2} + 2}{\frac{x-2}{x+2} - 2} = \frac{\frac{x-2+2x+4}{x+2}}{\frac{x-2-2x-4}{x+2}} = \frac{3x+2}{-x-6} = -\frac{3x+2}{x+6}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | $D_f = [-2, +\infty)$<br>$R_f = (-\infty, 2)$                                                                                                            |
| ۱۰ | $\sqrt{n} + \sqrt{y} = 2 \Rightarrow \sqrt{y} = 2 - \sqrt{n} \Rightarrow y = (2 - \sqrt{n})^2 = f(n)$<br>$f^{-1}(n) = (2 - \sqrt{n})^2$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | $D_f = [0, +\infty)$                                                                                                                                     |