الف) مسیاوونی ۱۲ اسپند ۷ تا ۹ $f(x)=x+[x]$

$y=x+[x] \to [y]=\big[x+[x]\big]$

$\to [y]=[x]+[x]=2[x] \to \underbrace{[x]}_{y-x}=\frac{1}{2}[y]$

$\to y-x=\frac{1}{2}[y] \to x=y-\frac{1}{2}[y] \to f^{-1}(x)=x-\frac{1}{2}[x]$

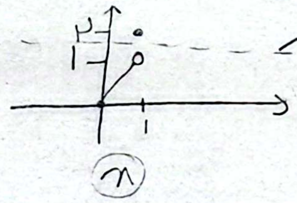
$f^{-1}(x)=f(x) \to x-\frac{1}{2}[x]=x+[x] \to \frac{3}{2}[x]=0 \to [x]=0$

$\to 0\leq x<1 \to$ بی‌شمار

ب) $f(f^{-1}(x))=x \to 2x-\frac{3}{2}=x \to x=\frac{3}{2}$

$x\in D_{f^{-1}}$

$x\in R_f$



$\to \frac{3}{2}\notin R_f$

۲-

مجانب قائم $\to 4x+b=0 \to x=-\frac{b}{4}$ مجانب افق $\to y=\frac{a}{4}$

$\to x=y \to -\frac{b}{4}=\frac{a}{4} \to a=-b \to$ تابع بر وارون خود منطبق است

$\to f(f(x)) \xrightarrow[f(x)=f^{-1}(x)]{}$ $f(f^{-1}(x))=x \to 5-\sqrt{4}$

۳-

$\frac{mx+3}{x+m+3}=x \to x^2+mx+3x=mx+3$

$\to x^2+3x-3=0 \quad \frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{-3}{-3}=1$

۴-

$|2x+5|-|5x-2|=$

$y=-3x+7 \to x=\frac{7-y}{3}$

$f^{-1}=\frac{7}{3}-\frac{x}{3}$

$-\frac{5}{2} \qquad \frac{2}{5} \qquad 2x+5-5x+2=-3x+7$

بازه نزولی

$-2x-5+5x-2$

$=3x-7$

$2x+5+5x-2=7x+3$

۵-

$$g(g^{-1}(x)) = g\left(af^{-1}\left(\frac{x+b}{c}\right)+d\right) \Rightarrow$$

$$x = 2f\left(3af^{-1}\left(\frac{x+b}{c}\right)+3d\right)-1$$

$$f\left(3af^{-1}\left(\frac{x+b}{c}\right)+3d\right) = \frac{x+1}{2} \rightarrow a=\frac{1}{3} \quad b=1 \quad c=2 \quad d=0$$

$$\frac{ac+d}{2b} = \frac{\frac{2}{3}}{2} = \frac{1}{3}$$

۶-

$$y = x+2\sqrt{x} = (\sqrt{x}+1)^2-1 \rightarrow (\sqrt{x}+1)^2 = y+1$$

$$\rightarrow \sqrt{x}+1 = \pm\sqrt{y+1} \rightarrow \sqrt{x} = \sqrt{y+1}-1 \rightarrow$$

$$x = y+1+1-2\sqrt{y+1} \rightarrow x = y+2-2\sqrt{y+1} \rightarrow y = x+2-2\sqrt{x+1}$$

$$D_{f^{-1}} = R_f \qquad R_f \rightarrow (\sqrt{x}+1)^2-1 \rightarrow$$

$$\sqrt{x} \geq 0 \rightarrow (\sqrt{x}+1)^2 \geq 1$$

$$D_{f^{-1}} = (0,+\infty)$$

$$\rightarrow (\sqrt{x}+1)^2-1 \geq 0$$

۷-

$$y = \underbrace{x-1}_{\text{صعودی اکید}} + \underbrace{2^x}_{\text{صعودی اکید}} \rightarrow 2^x+x-1 = x \rightarrow 2^x = 1 \rightarrow x=0$$

$$(0,0)$$

۸-

$$f^{-1}(x)-x \geq 0 \rightarrow f^{-1}(x) \geq x \xrightarrow{\text{از طرفین } f \text{ می‌گیریم}} \underbrace{f(f^{-1}(x))}_{x,\ x\in D_{f^{-1}}} \geq f(x)$$

$$\rightarrow f(x) \leq x \rightarrow x^3+4x \leq x \rightarrow x^3+3x \leq 0 \rightarrow x(x^2+3) \leq 0 \rightarrow x \leq 0$$

اعداد صحیح نامثبت

$0 \to 2$

$1 \to$

9

$f^{-1}(x+4)=x-3 \to f(x-3)=x+4 \qquad x \in D_{f(x-3)}$, $x \geq 0$

$\to -(x-3)+\sqrt{x+1} = x+4 \to \sqrt{x+1} = x+4+x-3$

$-x+3$

$\sqrt{x+1} = 2x+1 \to x+1 = 4x^2+4x+1$

$4x^2+3x=0 \to x(4x+3)=0 \to x=0$

$x=-\frac{3}{4}$ غ‌ق‌ق

۱۵ـ

$f(f(x)) =$

$\sqrt{y} = 2-\sqrt{x} \rightarrow y = (2-\sqrt{x})^2$

$f(f(x)) = f((2-\sqrt{x})^2) = \left(2-\sqrt{(2-\sqrt{x})^2}\right)^2 = (2-|2-\sqrt{x}|)^2$

چون $2-\sqrt{x}=\sqrt{y}$ پس $2-\sqrt{x}\geq 0$

$(2-2+\sqrt{x})^2 = (\sqrt{x})^2 = x$

$\sqrt{y} = 2-\sqrt{x} \rightarrow 2-\sqrt{x}\geq 0 \rightarrow x\leq 4$