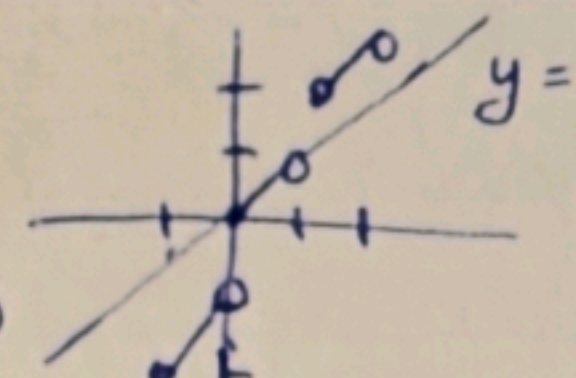


الف) $f^{-1} \circ f(x) \xrightarrow{\text{سمت چپ}} f^{-1}(f(x))$ (تابع اکسپانند در سمت چپ و سمت راست به ترتیب اول و دوم قطع می‌کنند)



ب) $f^{-1} \circ f(x) = x, x \in D_{f^{-1}} \rightarrow x = 2x - \frac{4}{f}$
 $\Rightarrow x = \frac{4}{f}, D_{f^{-1}} = \dots \cup [0, 1) \cup [2, 4) \cup \dots \Rightarrow$ جواب ندارد

1) $y = \frac{ax + 3}{x + b} \rightarrow \frac{a}{x} = \frac{-b}{x} = x \Rightarrow a = -b \Rightarrow y = \frac{ax + 3}{x - a}$

$a = -d \Rightarrow f(x) = f^{-1}(x) \rightarrow f(5 - \sqrt{4}) = 5 - \sqrt{4} \Rightarrow$

$f \circ f(5 - \sqrt{4}) = 5 - \sqrt{4}$

2) $f(x) = \frac{mx + 3}{x + m + 3} = x \rightarrow x^2 + mx + 3x = mx + 3$

$\rightarrow x^2 + 3x - 3 = 0 \quad \begin{cases} S = -3 \\ P = -3 \end{cases} \Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{S}{P} = 1$

$f(x) = |2x + 5| - |4x - 2| \xrightarrow{x > \frac{2}{4}} f(x) = 2x + 5 - 4x + 2$

$\Rightarrow -2x + 7 \Rightarrow f^{-1}(x) = \frac{7 - x}{2}$

• $g(x) = 2f(x) - 1 \rightarrow g^{-1}(x) \Rightarrow y = 2f(x) - 1$ (A)

$\frac{y+1}{2} = f(x) \xrightarrow{\text{سمت چپ}} f^{-1}\left(\frac{y+1}{2}\right) = x \rightarrow y = g^{-1}(x) = \frac{1}{2}f^{-1}\left(\frac{x+1}{2}\right)$

• $\frac{1}{2}f^{-1}\left(\frac{x+1}{2}\right) = a f^{-1}\left(\frac{x+b}{c}\right) + d \Rightarrow \begin{cases} a = \frac{1}{2} \\ b = 1 \\ c = 2 \\ d = 0 \end{cases} \Rightarrow \frac{ac+d}{2b} = \frac{\frac{1}{2} \cdot 2 + 0}{2 \cdot 1} = \frac{1}{2} = \frac{1}{2}$

$$f \circ g = x + \sqrt{x} \xrightarrow{\sqrt{x}=t} y = t^2 + \sqrt{x} \rightarrow t^2 + \sqrt{x} - y = 0 \rightarrow \quad (6)$$

$$\frac{-1 \pm \sqrt{1+y}}{2} = -1 \pm \sqrt{1+y} \xrightarrow{x>} -1 + \sqrt{1+y} \xrightarrow{t^2=x} f^{-1}(x) = (\sqrt{1+x} - 1)^2$$

$$y = x - 1 + \sqrt{x} : \text{اگرچه صعودی} \Rightarrow \text{دارد خود را در خودش می‌تاباند} \Rightarrow \quad (7)$$

تا به اول و دوم قطع می‌کند

$$x - 1 + \sqrt{x} = x \rightarrow \sqrt{x} = 1 \Rightarrow x = 0 \rightarrow \text{در یک نقطه قطع می‌کند}$$

$$y = \sqrt{f^{-1}(x) - x} \rightarrow f^{-1}(x) \geq x, \quad f(x) = x^3 + \epsilon x \quad (8)$$

$$\Rightarrow f^{-1}(x) = \text{اگرچه صعودی} \Rightarrow x \geq f(x) \Rightarrow x \geq x^3 + \epsilon x \Rightarrow x(x^2 + \epsilon) \leq 0$$

$$\Rightarrow x \leq 0 \Rightarrow D_{f^{-1}} = (-\infty, 0] \Rightarrow \text{در یک مجموعه نامنفی (مفرد)}$$

$$y = f(x) = -x + \sqrt{x + \epsilon^2} \rightarrow x + \epsilon^2 = -y + \sqrt{y + \epsilon^2} \quad y \geq -\epsilon^2 \quad (9)$$

$$\xrightarrow{y + \epsilon^2 = t^2} -t^2 + t = x \rightarrow t^2 - t + x = 0 \rightarrow \quad \Rightarrow x \geq \epsilon^2$$

$$t > \frac{1 + \sqrt{1 - 4\epsilon x}}{2} = t$$

$$\xrightarrow{t >} \left(\frac{1 + \sqrt{1 - 4\epsilon x}}{2} \right)^2 = y + \epsilon^2 \rightarrow \left(\frac{1 + \sqrt{1 - 4\epsilon x}}{2} \right)^2 - \epsilon^2 = x - \epsilon^2 \Rightarrow$$

$$\left(\frac{1 + \sqrt{1 - 4\epsilon x}}{2} \right)^2 = x + 1 \Rightarrow \epsilon x + \epsilon^2 = 1 + 1 - \epsilon x + 2\sqrt{1 - 4\epsilon x} \Rightarrow 2\epsilon x + \epsilon^2 = 2\sqrt{1 - 4\epsilon x}$$

$$\Rightarrow 14\epsilon x^2 + 12\epsilon x = 0 \rightarrow \epsilon x(\epsilon x + 3) = 0 \quad \left(-\frac{3}{\epsilon} \right) \Rightarrow \text{بر خود} \quad (10)$$

$$\sqrt{y} = 1 - \sqrt{x} \rightarrow y = 1 - 2\sqrt{x} + x \rightarrow f \circ f = \quad (10)$$

$$1 - \epsilon \in \sqrt{(1 - \sqrt{x})^2} + 1 - \epsilon \sqrt{x} + x \xrightarrow{1 - \sqrt{x} > 0} 1 - 1 + \epsilon \sqrt{x} + 1 - \epsilon \sqrt{x} + x$$

$$f \circ f(x) = x, \quad D_f = \{x \in D_f, f(x) \in D_f\} \Rightarrow D_f = [0, 1]$$

$(-\infty, 1] \in [0, +\infty)$

