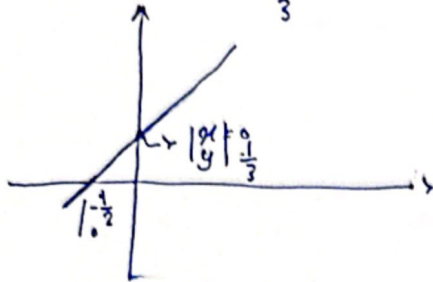


$$2y = 3x - 1 \rightarrow x = \frac{2y+1}{3} \rightarrow f^{-1}(x) = \frac{2x+1}{3}$$



$$S = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

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چون بازه باز است یک به یک نیست
 $y = (x-1)^2 + 2 \rightarrow y-2 = (x-1)^2$
 $\sqrt{y-2} = x-1 \rightarrow f^{-1}(x) = \sqrt{x-2} + 1$

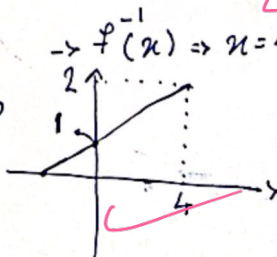
چون بازه بسته است یک به یک است
 $S = \frac{2}{2} = 1 \rightarrow f^{-1}(x) = \sqrt{x-2} + 1$
 $x \in [2, +\infty)$
 $\sqrt{x-2} + 1 \geq 3 \rightarrow \sqrt{x-2} \geq 2$
 $x-2 \geq 4 \rightarrow x \geq 6$
 $x \in [6, +\infty)$

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$$2x - \sqrt{16 - 16x + 4x^2} = 2x - \sqrt{(2x-4)^2} \rightarrow 2x - |2x-4|$$

$$\begin{array}{r|l} 2 & 4 \\ \hline 4x-4 & 4 \end{array}$$

دویندگی نیست



$$\rightarrow f^{-1}(x) \Rightarrow x = 4y - 4 \rightarrow y = \frac{x+4}{4}, x \in (-\infty, +4)$$

۲

$$\frac{x_1}{\sqrt{x_1^2-5}} = \frac{x_2}{\sqrt{x_2^2-5}} \Rightarrow x_1 \sqrt{x_2^2-5} = x_2 \sqrt{x_1^2-5} \xrightarrow{\wedge 2} x_1^2 x_2^2 - 5x_1^2 = x_2^2 x_1^2 - 5x_2^2$$

دو طرف را در صورت یکدیگر ضرب می‌کنیم و علامت‌ها را می‌بینیم
 $\rightarrow -5x_1^2 = -5x_2^2 \rightarrow x_1^2 = x_2^2 \rightarrow x_1 = \pm x_2$
 $\frac{y}{\sqrt{y^2-5}} = x \rightarrow x \sqrt{y^2-5} = y \xrightarrow{\wedge 2} x^2 y^2 - 5x^2 = y^2 \rightarrow -5x^2 = y^2(1-x^2)$
 $y^2 = \frac{-5x^2}{1-x^2} \rightarrow y = \sqrt{\frac{-5x^2}{1-x^2}}$

۲

$$f(x) \begin{cases} x^2 - 4x + k & x \geq 2 \\ -x^2 + 6x + 3k & x < 2 \end{cases} \rightarrow S: 2, \mathbb{R}: [k-4, +\infty)$$

$$S: 3, \mathbb{R}: (-\infty, 8+3k)$$

که چون شامل در نیست

$$\emptyset = [k-4, +\infty) \cap (-\infty, 8+3k) \text{ باشد}$$

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$$f(x): x > 0 \rightarrow 4x \quad f^{-1}(x): x > 0 \quad \frac{x}{4}$$

$$f(x): x < 0 \rightarrow 2x \quad f^{-1}(x): x < 0 \quad \frac{x}{2}$$

$$\frac{3}{8}x + b|x| \rightarrow \frac{3}{8}x + b|x| = \frac{2x}{8} \rightarrow b = -\frac{1}{8}$$

$$ab = -\frac{3}{64}$$

بین ضرب x میانیلیں کی لیں

$$\frac{\frac{1}{4} + \frac{1}{2}}{2} = \frac{3}{8}$$

(2)

6

$$\frac{3(2) + 4}{x + m} = -1 \rightarrow -\log x - b m = 10 \rightarrow m = -3 \quad f^{-1}(x) = x = \frac{3y + 4}{y - 3} = -\frac{3x - 4}{1x - 3}$$

$$= \frac{3x + 4}{x - 3} \quad f \circ f^{-1}(x) = x$$

$$f \circ f^{-1}(x) + f^{-1}(x) = x \rightarrow x + \frac{3x + 4}{x - 3} = x \rightarrow \frac{3x + 4}{x - 3} = 0$$

$$3x + 4 = 0 \rightarrow x = -\frac{4}{3}$$

(2)

7

$$f(a) = -2 \quad f(a) = 2 - g\left(\frac{x}{2}\right) = -2 \rightarrow g\left(\frac{x}{2}\right) = 4 \quad \frac{x}{2} = b$$

$$g(b) = 4 \quad a = 3 - 2x$$

$$\rightarrow 4b + 0 = 2x + 3 - 2x = 3$$

(2)

8

$$y = \frac{5x - 13}{1 - x} \quad \text{نسبت؟ بعد از ضربی کنیم} \quad y' = \left(\frac{-5x - 13}{1 + x} \right) = \frac{5x + 13}{1 + x} \quad \text{در دو طرف را ضرب کنیم} \quad \frac{5(x - 2) + 13}{1 + (x - 2)} = \frac{5x + 3}{x - 1}$$

$$\frac{5x + 3}{x - 1} - 3 \rightarrow \frac{2x + 6}{x - 1} = f(x) \rightarrow f^{-1}(x) = \frac{-x - 6}{x - 2} = \frac{x + 6}{x - 2}$$

$$\frac{x + 6}{x - 2} = \frac{2x + 6}{x - 1} \Rightarrow x^2 + 5x - 6 = 2x^2 + 2x - 12 \rightarrow x^2 - 3x - 6 = 0 \quad \Delta = 33$$

$$x = \frac{3 + \sqrt{33}}{2} \quad \frac{3 - \sqrt{33}}{2} \rightarrow \frac{3 + \sqrt{33}}{2} + \frac{3 - \sqrt{33}}{2} = \frac{6}{2} = 3$$

(2)

9

$$y = f^{-1}(x) \Rightarrow y = x + 4[x] \quad \text{اور این کوئی لیں} \quad [y] = [x + 4[x]] \quad \text{در دو طرف 4 ضرب کنیم} \quad [y] = [x] + 4[x]$$

$$[y] = 5[x] \rightarrow \frac{1}{5}[y] = [x] \quad \text{نسبت؟ بعد از ضربی کنیم} \quad \frac{1}{5}[y] = \frac{y - x}{4} \Rightarrow 4[y] = 5y - 5x$$

$$\rightarrow 5x = 5y - 4[y] \rightarrow x = \frac{5y - 4[y]}{5} \quad y = x - \frac{4}{5}[x]$$

$$(f \circ g)(x) = x + 4[x] - x - \frac{4}{5}[x] = \frac{16}{5}[x]$$

(1, 1, 0)

10

$$f^{-1}(x) = g(x) = x - .1[x] \rightarrow f \circ g = x + 5[x] - x + .1[x] = 5.1[x]$$