برای محیا روشن

۱)

$2x = 3y - 1$

$\Rightarrow y = \frac{2x+1}{3}$

$x=0 \rightarrow y = \frac{1}{3}$

$y=0 \rightarrow x = -\frac{1}{2}$

$S = \frac{\frac{1}{3} \times \frac{1}{2}}{2} = \frac{1}{12}$

۲) الف)

[illegible]

$\frac{-b}{2a} = \frac{2}{2} = 1$

$y = (x-1)^2 + 2$

$x = (y-1)^2 + 2$

$y = \oplus\sqrt{x-2} + 1$

$D = [2, \infty)$

ب)

$x=3 \rightarrow 9 - 6 + 3 = 6$

$y = \sqrt{x-2} + 1$

$D = [6, \infty)$

۳)

$f(x) = 2x - |2x-4|$

$x \geq 2: \; 4$

$4x - 4$ ✓

$f^{-1}(x) = \frac{x+4}{4}$ $\quad S = 1 \times 4 \times \frac{1}{2} = 2$

۴)

$f(x_1) = f(x_2)$

$\frac{x_1}{\sqrt{x_1^2-5}} = \frac{x_2}{\sqrt{x_2^2-5}} \rightarrow \frac{x_1^2}{x_1^2-5} = \frac{x_2^2}{x_2^2-5}$

$x_1^2x_2^2 - 5x_1^2 = x_1^2x_2^2 - 5x_2^2 \Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = \oplus x_2 \Rightarrow x_1 = x_2$

غ ق ق

$x = \frac{y}{\sqrt{y^2-5}} \rightarrow x^2 = \frac{y^2}{y^2-5} \rightarrow y^2x^2 - 5x^2 = y^2$

$y^2x^2 - y^2 = 5x^2 \rightarrow y^2 = \frac{5x^2}{x^2-1} \rightarrow y = \pm\sqrt{\frac{5x^2}{x^2-1}}$

$\rightarrow y = \frac{\sqrt{5}\,x}{\sqrt{x^2-1}}$

۵)

$x=2 \rightarrow 8 + 3K \leq K$

$\Rightarrow K \leq -4$

۶)

$f(x) = \begin{cases} 4x & x \geq 0 \\ 2x & x < 0 \end{cases}$

$f^{-1}(x) = \begin{cases} \frac{x}{4} & x \geq 0 \\ \frac{x}{2} & x < 0 \end{cases}$

$\frac{\frac{1}{2} + \frac{1}{4}}{2} = \frac{3}{8}$ $\quad \frac{1}{2} - \frac{3}{8} = \frac{1}{8}$

$g(x) = \frac{3}{8}x - \frac{1}{8}|x|$

$ab = \frac{3}{8} \times -\frac{1}{8} = -\frac{3}{64}$

(۷)

$$-10 = \frac{۳\times۲+۴}{۲+m}$$

$$\Rightarrow m = -۳$$

$$\Rightarrow f(x) = f^{-1}(x)$$

$$y = x + \frac{۳x+۴}{x-۲} = x$$

$$\Rightarrow x = -\frac{۴}{۳}$$

(۹)

$$-y = \frac{-۵x-۱۳}{۱+x}$$

$$y = \frac{۵(x-۲)+۱۳}{۱+x-۲} \quad -۲$$

$$= \frac{۵x-۳-۳x+۲}{x-۱} = \frac{۲x}{x-۱}$$

$$\frac{۲x}{x-۱} = \frac{-x}{x-۲}$$

$$۲x^۲ - ۴x = -x^۲ + x$$

$$۳x^۲ - ۵x = ۰ \qquad \frac{-b}{a} = \frac{۵}{۳}$$

(۱۰)

$$x = y + ۴[y]$$

$$[x] = [y] + ۴[y]$$

$$[x] = ۵[y]$$