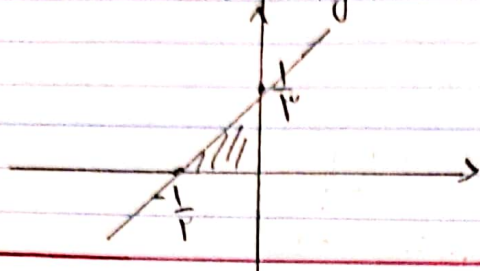


مهرساختی / تکلیف شماره ۱۵

$$2y = 3x - 1 \rightarrow 3x = 2y + 1 \rightarrow x = \frac{2y+1}{3} \rightarrow f^{-1} = \frac{2x+1}{3} \rightarrow \begin{vmatrix} 0 & -\frac{1}{3} \\ \frac{1}{3} & 0 \end{vmatrix} \quad (1)$$



$$S = \frac{\frac{1}{3} \times \frac{1}{3}}{2} = \frac{1}{12}$$

(الف) $y = 2x^2 - 2x + \frac{3}{2}, x \in [1, +\infty) \rightarrow x = \frac{-b}{2a} = \frac{1}{2} = 1$

$$y = 1 - 2 + \frac{3}{2} = \frac{1}{2}$$

$$y = (x-1)^2 + \frac{1}{2} \rightarrow y - \frac{1}{2} = (x-1)^2 \rightarrow \sqrt{y - \frac{1}{2}} = |x-1| \rightarrow$$

$$x = \sqrt{y - \frac{1}{2}} + 1 \rightarrow f^{-1} = \sqrt{2x - 1} + 1; x \in [\frac{3}{2}, +\infty)$$

(ب) $y = 2x^2 - 2x + \frac{3}{2}, x \in [\frac{3}{2}, +\infty) \rightarrow x = \frac{-b}{2a} = \frac{1}{2} = 1$

$$x = \frac{3}{2} \rightarrow y = \frac{9}{2} - 3 + \frac{3}{2} = \frac{3}{2}$$

$$y = 1 - 2 + \frac{3}{2} = \frac{1}{2}$$

$$y = (x-1)^2 + \frac{1}{2} \rightarrow y - \frac{1}{2} = (x-1)^2 \rightarrow$$

$$\sqrt{y - \frac{1}{2}} = |x-1| \rightarrow x = \sqrt{y - \frac{1}{2}} + 1 \rightarrow f^{-1} = \sqrt{2x - 1} + 1; x \in [\frac{3}{2}, +\infty)$$

$$Rf = [\frac{3}{2}, +\infty) = D_{f^{-1}}$$

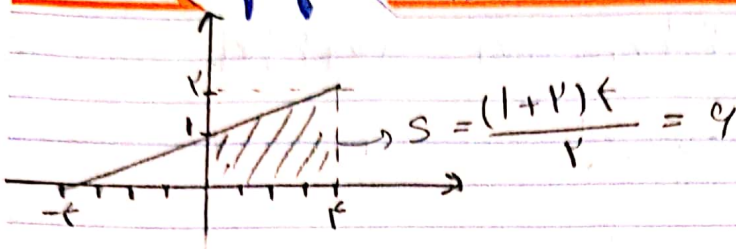
$$f(x) = 2x^2 - \sqrt{14 - 4x(x-1)} = 2x^2 - \sqrt{4x^2 - 14x + 14} = \quad (3)$$

$$2x^2 - |2x^2 - 4|$$

$$(2x^2 - 4)^2$$

$$\begin{cases} x \geq 2, & y = 4 \rightarrow \text{دارون (نیز نیست)} \\ x < 2, & 4x - 4 \end{cases}$$

$$y = 4x - 4 \rightarrow x = \frac{y+4}{4} \rightarrow f^{-1} = \frac{x+4}{4} \rightarrow \begin{vmatrix} 0 & -4 \\ 1 & 0 \end{vmatrix} \quad \frac{4}{4}$$



$$y = \frac{x}{\sqrt{x^2 - \omega}} \quad y_1 = y_r \rightarrow \frac{x_1}{\sqrt{x_1^2 - \omega}} = \frac{x_r}{\sqrt{x_r^2 - \omega}} \rightarrow$$

(۴)

$$\frac{x_1^2}{x_1^2 - \omega} = \frac{x_r^2}{x_r^2 - \omega} \rightarrow \cancel{x_1^2 x_r^2} - \omega x_1^2 = \cancel{x_1^2 x_r^2} - \omega x_r^2 \rightarrow$$

$$x_1^2 = x_r^2 \rightarrow x_1 = \pm x_r \quad \text{مستویات } y > x \rightarrow x_1 = x_r \rightarrow \text{پایین مستوی}$$

$$y = \frac{x}{\sqrt{x^2 - \omega}} \rightarrow x = \frac{-\omega y}{y^2 - 1} = \frac{\omega y}{y^2 - 1} \rightarrow x = \pm \sqrt{\frac{\omega y}{y^2 - 1}}$$

$$f(m) = \sqrt{\frac{\omega x^2}{x^2 - 1}}$$

$$g(x) = \begin{cases} x^2 - x + k & x \geq 1 \rightarrow -\frac{b}{2a} = 1 \\ -x^2 + 4x + 3k & x < 1 \rightarrow \frac{b}{2a} = 1 \end{cases}$$

(۵)

$$1) x = 1 \rightarrow y = 1 - 1 + k = k - 1$$

$$2) x = 1 \rightarrow -1 + 4 + 3k = 3k + 1$$

$$3k + 1 < k - 1 \rightarrow 2k < -2 \rightarrow k < -1$$

$$f(x) = 3x + 1$$

$$f^{-1}(x) = g(x) = ax + b$$

$$\begin{cases} 3x & x \geq 0 \rightarrow y = 3x \rightarrow x = \frac{y}{3} \\ 2x & x < 0 \rightarrow y = 2x \rightarrow x = \frac{y}{2} \end{cases}$$

$$f^{-1} = \begin{cases} \frac{x}{3} & x \geq 0 \\ \frac{x}{2} & x < 0 \end{cases} \rightarrow \begin{cases} x(a+b) & x \geq 0 \\ x(a+b) & x < 0 \end{cases}$$

$$\begin{cases} a+b = \frac{1}{3} \\ a-b = \frac{1}{2} \end{cases}$$

$$a \cdot b = \frac{3}{1} \times -\frac{1}{1} = -\frac{3}{1}$$

$$3a = \frac{3}{1} \rightarrow a = \frac{3}{1}, b = \frac{1}{1}$$

$$f(x) = \frac{3x+1}{x+1} \quad \left| \begin{matrix} 1 \\ -1 \end{matrix} \right. \quad y = f \circ f^{-1}(x) + f^{-1}(x)$$

⑤

$$\left| \begin{matrix} x=1 \\ y=1 \end{matrix} \right. \quad -1 = \frac{1+1}{1+1} \rightarrow -1 = 1 \rightarrow -1 = 1 \rightarrow m = -1$$

$$f(x) = \frac{3x+1}{x-1} \rightarrow x = -\frac{-y-1}{y-1} = \frac{y+1}{y-1}$$

$$f^{-1}(x) = \frac{3x+1}{x-1} \quad y = x + f^{-1}(x) = x + \frac{3x+1}{x-1}$$

$$\text{نیمه اول سال} \rightarrow y = x \Rightarrow x + \frac{3x+1}{x-1} = x \Rightarrow \frac{3x+1}{x-1} = 0 \Rightarrow x = -\frac{1}{3}$$

$$f(3-2x) = 2 - g\left(\frac{x}{2}\right) \quad f g^{-1}(x) + f^{-1}(-x)$$

⑧

$$\begin{cases} f(3-2x) = -1 \\ g\left(\frac{x}{2}\right) = 2 \end{cases} \rightarrow -1 = 2 - x \rightarrow \checkmark$$

$$f\left(\frac{x}{2}\right) + 3 - 2x = 2x + 3 - 2x = 3$$

$$y = \frac{\omega x - 13}{1 - x} \xrightarrow{\text{قرینه}} -x = \frac{-\omega y - 13}{1 + y} \rightarrow x = \frac{\omega y + 13}{1 + y}$$

⑨

$$y = -\frac{x - 13}{x - \omega} = \frac{13 - x}{x - \omega} \xrightarrow{\text{تغییر متغیر}} y = \frac{13 - (x - 2)}{x - 2 - \omega} = \frac{15 - x}{x - \nu}$$

$$\xrightarrow{\text{تغییر متغیر}} y = \frac{15 - x}{x - \nu} - 2 = \frac{-x + 30}{x - \nu} = f(x)$$

$$f^{-1}(x) = -\frac{-\nu x - 30}{x + 2} = \frac{\nu x + 30}{x + 2}$$

$$\frac{\nu x + 30}{x + 2} = \frac{30 - x}{x - \nu} \rightarrow \nu x^2 - x + 30x - 2\omega^2 = 30x - x + 15 - 2x^2 - 15x$$

$$\rightarrow 11x^2 - 33x - 30 = 0 \xrightarrow{\div 11} x^2 - 3x - 30 = 0$$

$$\text{مجموع جملات} = -\frac{b}{a} = \frac{3}{1} = 3$$

$$f(x) = x + r[x]$$

$$f(x) - g(x) = ?$$

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$$\rightarrow y = x + r[x] \quad [] \quad [y] = [x + r[x]] = [x] + r[x] \quad \text{قرینه}$$

$$[x] = \omega[y] \rightarrow [y] = \frac{[x]}{\omega} \rightarrow x - \frac{r[x]}{\omega} = y = g(x) \quad \omega[x]$$

$$x = y + r[y]$$

$$f(x) - g(x) = x + r[x] - x + \frac{r[x]}{\omega} = \frac{r[x]}{\omega}$$