

تکلیف شماره ۱

بسم خدا

۱۹، ۷۵ آفرین

در ادامه دقت

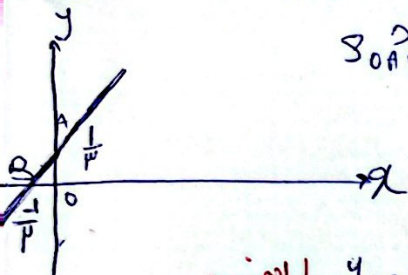
$$2y = 3x + 1 \Rightarrow 2y + 1 = 3x \Rightarrow x = \frac{2y+1}{3} \Rightarrow f^{-1}(y) = \frac{2y+1}{3}$$

①

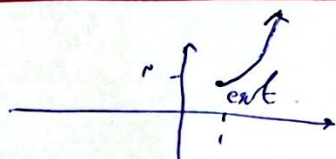
$$f(x) = 3x + 1 \Rightarrow y = 3x + 1 \Rightarrow y - 1 = 3x \Rightarrow x = \frac{y-1}{3}, f^{-1}(y) = \frac{y-1}{3}$$

②

$$S_{OAB} = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{18}$$



الف) $y = x^2 - 2x + 3, x \in [1, +\infty)$



③

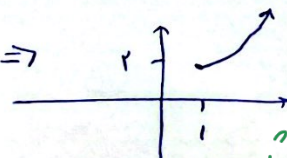
$$y = x^2 - 2x + 3 = (x-1)^2 + 2 \Rightarrow y - 2 = (x-1)^2 \Rightarrow x-1 = \sqrt{y-2} \Rightarrow x = \sqrt{y-2} + 1 \Rightarrow f^{-1}(y) = \sqrt{y-2} + 1$$

$$(x-1)^2 = y-2 \Rightarrow x-1 = \sqrt{y-2} \Rightarrow x = \sqrt{y-2} + 1 \Rightarrow f^{-1}(y) = \sqrt{y-2} + 1$$

$$D_{f^{-1}}(y) = R_f = [2, +\infty)$$

ب) $y = x^2 - 2x + 3, x \in [3, +\infty)$

$$S_n = \frac{-b}{2a} = \frac{2}{2} = 1 \Rightarrow$$



$$D_f = R_f = [4, +\infty)$$

مواضع باشد $x \geq 3$

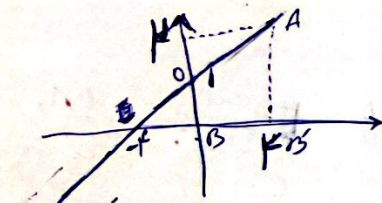
$$f^{-1}(y) = \sqrt{y-2} + 1$$

$$D_{f^{-1}}(y) = [2, +\infty)$$

$$f(x) = 2x - \sqrt{14 - 8x + (x-5)^2} = 2x - \sqrt{14 - 8x + x^2 - 10x + 25} = 2x - \sqrt{x^2 - 18x + 39} = 2x - \sqrt{(x-9)^2 - 72}$$

④

$$f(x) = \begin{cases} x & x \geq 2 \\ 2x - 5 & x < 2 \end{cases} \Rightarrow f^{-1}(y) = \begin{cases} y & y \geq 2 \\ \frac{y+5}{2} & y < 2 \end{cases}$$



$$S_{OAB} = \dots$$

$$\frac{(1+2) \times 5}{2} = \frac{15}{2}$$

$$R_f = D_{f^{-1}} = x \leq 2$$

$$y = \frac{x}{\sqrt{x^2 - a}} \Rightarrow y \sqrt{x^2 - a} = x \Rightarrow y^2 (x^2 - a) = x^2$$

$$y^2 x^2 - a y^2 = x^2 \Rightarrow a y^2 = x^2 (y^2 - 1)$$

$$\Rightarrow x^2 = \frac{a y^2}{y^2 - 1} \Rightarrow x = \pm \frac{y \sqrt{a}}{\sqrt{y^2 - 1}} \Rightarrow f'(x) = \frac{x \sqrt{a}}{\sqrt{x^2 - a}}$$

فقط مقدار قابل قبول است زیرا x دل هم علامت اند

↓ بررسی کنید بول تابع :

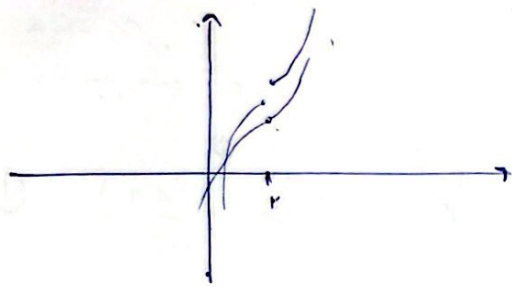
$$\text{if } (x_1, y) \text{ و } (x_2, y) \Rightarrow x_1 = x_2 \Rightarrow y = \frac{x_1}{\sqrt{x_1^2 - a}} = \frac{x_2}{\sqrt{x_2^2 - a}} \Rightarrow$$

$$\frac{x_1^2}{x_1^2 - a} = \frac{x_2^2}{x_2^2 - a} \Rightarrow x_1^2 x_2^2 - a x_1^2 = x_1^2 x_2^2 - a x_2^2 \Rightarrow$$

$$a x_1^2 = a x_2^2 \Rightarrow x_1 = \pm x_2 \Rightarrow \boxed{x_1 = x_2}$$

مقدار متغیر قابل قبول زیرا x_1 و x_2 هم علامت اند

تابع یک به یک است



$$f(x) = \begin{cases} x^2 - \varepsilon x + k & x \geq r \\ -x^2 + 4x + 3k & x < r \end{cases} \quad (3)$$

$\Rightarrow$ $f(r) = 0 \Rightarrow -r^2 + 4r + 3k = 0 \Rightarrow 3k = r^2 - 4r \Rightarrow k = \frac{r^2 - 4r}{3} = \frac{r}{3}(r - 4)$

$$r = 2 \Rightarrow -\varepsilon + 12 + 3k \leq \varepsilon - 1 + k \Rightarrow 3k + 11 \leq k - \varepsilon \Rightarrow 2k \leq -\varepsilon - 11 \Rightarrow k \leq -\frac{\varepsilon + 11}{2}$$

$$f(x) = \varepsilon x + |x| \Rightarrow f(x) = \begin{cases} \varepsilon x & x \geq 0 \\ |x| & x < 0 \end{cases} \Rightarrow f^{-1}(x) = \begin{cases} \frac{x}{\varepsilon} & x \geq 0 \\ \frac{x}{1} & x < 0 \end{cases} \quad (4)$$

$$a = \frac{\frac{x}{\varepsilon} + \frac{x}{1}}{1} = \frac{\varepsilon x}{\varepsilon + 1} \Rightarrow f^{-1}(x) = \frac{\varepsilon x}{\varepsilon + 1} + \frac{|x|}{1} \iff f^{-1}(x) = g(x) = ax + b|x|$$

$a = \frac{\varepsilon}{\varepsilon + 1}, b = \frac{1}{\varepsilon + 1} \Rightarrow a + b = \frac{\varepsilon + 1}{\varepsilon + 1} = 1$

$$f(x) = \frac{\varepsilon x + \varepsilon}{\varepsilon + 1} \Rightarrow \frac{1}{\varepsilon + 1} = -1 \Rightarrow -\varepsilon - 1 = 1 \Rightarrow -\varepsilon = 2 \Rightarrow \varepsilon = -2$$

$m = -\varepsilon = 2$

$$y = f \circ f^{-1}(x) + f^{-1}(x) = x \Rightarrow f^{-1}(x) = 0 \Rightarrow \frac{\varepsilon x + \varepsilon}{\varepsilon + 1} = 0 \Rightarrow \varepsilon x + \varepsilon = 0 \Rightarrow x = -1$$

$a + b = 0 \Rightarrow -1$

الفرضية
 $f^{-1}(x)$
 هي
 هي

$$f(3-x) = 2 - g\left(\frac{x}{2}\right)$$

$$\varepsilon g^{-1}(\varepsilon) + f^{-1}(-2) = ?$$

$$\varepsilon t + 3 - 2t = \textcircled{3}$$

جواب

$$g^{-1}(\varepsilon) = t \Rightarrow g(t) = \varepsilon$$

$$g\left(\frac{2t}{2}\right) = \varepsilon \Rightarrow x = 2t$$

$$f(3-x) = 2 - g(t) = 2 - \varepsilon = \textcircled{-2}$$

$$f(3-x) = -2 \Rightarrow f(3-2t) = -2$$

$$y = \frac{\Delta x - 13}{1-x} \xRightarrow{\substack{x \rightarrow -x \\ y \rightarrow -y}} y = - \frac{-\Delta x - 13}{x+1} = \frac{\Delta x + 13}{x+1} \xrightarrow{\substack{\text{نقص و حذف} \\ x \rightarrow x-2}} \quad (9)$$

$$y' = \frac{\Delta(x-2) + 13}{(x-2)+1} = \frac{\Delta x + 13}{x-1} \xRightarrow{\text{نقص و حذف}} y' = \frac{\Delta x + 13}{x-1} - 13 = \frac{\Delta x + 13 - 13x + 13}{x-1} = \frac{2x+4}{x-1} = f(x)$$

$$x = \frac{y+4}{y-2} \Rightarrow f^{-1}(x) = \frac{x+4}{x-2} \Rightarrow f(x) = f^{-1}(x) \Rightarrow \frac{2x+4}{x-1} = \frac{x+4}{x-2} \Rightarrow (10)$$

$$2x^2 - 5x + 4x - 12 = x^2 + \Delta x - 4 \Rightarrow x^2 - 13x - 4 = 0 \Rightarrow x = \frac{-b}{a} = 13$$

$$y = f(x) = x + \varepsilon[x]$$

$$\begin{array}{l} \text{قریبی نسبت به خط} \\ y=x \end{array} \Rightarrow \begin{array}{l} \text{هناں دلوں کا مجموعہ} \\ \text{بہ نسبت بہ خط یوں} \end{array} = g(x) \quad (11)$$

$$(f-g)(x) = ?$$

$$\begin{array}{l} \text{از تابعین کے اوقات میں} \\ \text{میں} \end{array} \rightarrow [y] = [x + \varepsilon[x]] = [x] + \varepsilon[x] = \Delta[x] \quad (12)$$

$$y = x + \varepsilon[x] \Rightarrow \frac{\varepsilon[x]}{\Delta} = \frac{\varepsilon[y]}{\Delta} \Rightarrow y = x + \varepsilon \frac{\varepsilon[y]}{\Delta} \Rightarrow x = y - \frac{\varepsilon[y]}{\Delta} \Rightarrow f^{-1}(x) = x - \frac{\varepsilon[x]}{\Delta} = g(x)$$

$$(f-g)(x) = f(x) - g(x) = x + \varepsilon[x] - x - \frac{\varepsilon[x]}{\Delta} = \varepsilon[x]$$