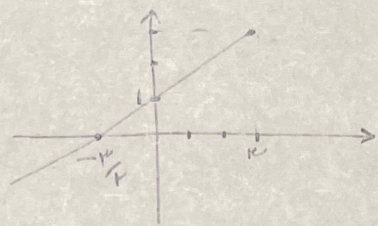


سوال ①

$$2y = 3x - 1 \xrightarrow{\text{دارن}} 3x = 2y + 1 \Rightarrow x = \frac{2y+1}{3} \Rightarrow y = \frac{3x+1}{3} \Rightarrow y = \frac{2}{3}x + \frac{1}{3}$$



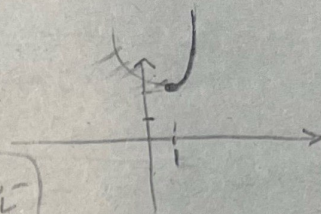
$$y=0 \Rightarrow 0 = \frac{2}{3}x + \frac{1}{3} \Rightarrow \frac{2}{3}x = -\frac{1}{3} \Rightarrow x = -\frac{1}{2}$$

$$x=0 \Rightarrow y = \frac{1}{3} \quad S = \frac{\frac{1}{3} \times \frac{1}{3}}{2} = \boxed{\frac{1}{12}}$$

سوال ②

الف) $y = x^2 - 2x + 2 \quad x \in [1, +\infty)$

$$\begin{cases} xS = -\frac{b}{2a} = \frac{2}{2} = 1 \\ yS = 2 \end{cases} \Rightarrow \boxed{\text{نیمه باز است}}$$

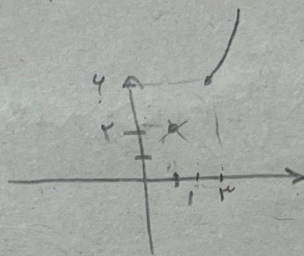


$$y = (x-1)^2 + 2 \xrightarrow{\text{دارن}} y - 2 = (x-1)^2 \xrightarrow{\text{جذر}} \sqrt{y-2} = x-1 \Rightarrow x = \sqrt{y-2} + 1$$

$$\Rightarrow \text{دارن} \quad y = \sqrt{x-2} + 1 \quad D_{f^{-1}} = R_f = [2, +\infty)$$

ب) $y = x^2 - 2x + 2 \quad x \in [2, +\infty)$

$$\begin{cases} xS = -\frac{b}{2a} = \frac{2}{2} = 1 \\ yS = 2 \end{cases} \quad \begin{cases} x=2 \\ y=4 \end{cases}$$



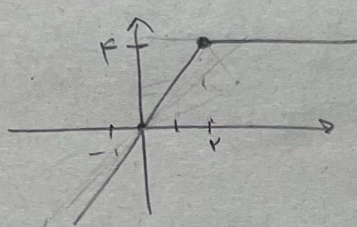
نیمه باز است

$$\text{دارن} \quad y = \sqrt{x-2} + 1 \quad D_{f^{-1}} = R_f = [4, +\infty)$$

$$f(x) = 2x - \sqrt{14 - 4x + x^2} \Rightarrow f(x) = 2x - \sqrt{x^2 - 4x + 14}$$

سوال ③

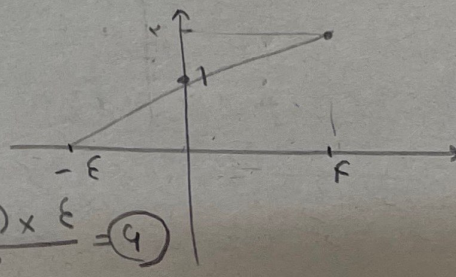
$$\Rightarrow f(x) = 2x - \sqrt{(x-2)^2} = 2x - |x-2|$$



نیمه باز است $(-\infty, 2]$

$$f^{-1} = \frac{x+2}{2} = \boxed{\frac{x}{2} + 1}$$

$$\begin{cases} 2x - 2x + 2 & x \geq 2 \rightarrow \text{نیمه باز است} \\ \frac{2x + 2x - 2}{2} & x < 2 \end{cases}$$



$$S = \frac{(1+2) \times 4}{2} = \boxed{6}$$

$$y = \frac{x}{\sqrt{x^2 - a}}$$

سؤال ٤

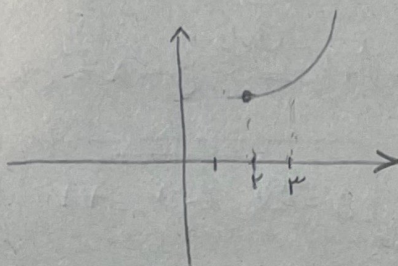
$$f(x_1) = f(x_2) \Rightarrow \frac{x_1}{\sqrt{x_1^2 - a}} = \frac{x_2}{\sqrt{x_2^2 - a}} \xrightarrow{x_1 \neq x_2} \frac{x_1^2}{x_1^2 - a} = \frac{x_2^2}{x_2^2 - a} \Rightarrow$$

$$x_1^2 x_2^2 - a x_1^2 = x_2^2 x_1^2 - a x_2^2 \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2 \Rightarrow \text{مربعين}$$

$$f^{-1} \text{ لـ } y = \frac{x}{\sqrt{x^2 - a}} \Rightarrow x = \frac{y}{\sqrt{y^2 - a}} \Rightarrow x^2 = \frac{y^2}{y^2 - a} \Rightarrow y^2 x^2 - a x^2 = y^2 \Rightarrow$$

$$y^2 x^2 - y^2 = a x^2 \Rightarrow y^2 = \frac{a x^2}{x^2 - 1} \Rightarrow y = \pm \sqrt{\frac{a x^2}{x^2 - 1}} \Rightarrow y = \frac{\sqrt{a} x}{\sqrt{x^2 - 1}}$$

$$g(x) = \begin{cases} x^2 - 4x + k & x \geq 2 \\ -x^2 + 4x + 3k & x < 2 \end{cases}$$



سؤال ٥

$$\begin{cases} x^2 - 4x + k \xrightarrow{x=2} -4 + k \\ -x^2 + 4x + 3k \xrightarrow{x=2} 4 + 3k \end{cases} \Rightarrow \begin{cases} -4 + k > 4 + 3k \\ -4 + k \leq 4 + 3k \end{cases} \Rightarrow \begin{cases} k < -4 \\ k \geq -4 \end{cases} \Rightarrow k = -4$$

$$f(x) = rx + |x| \quad g(x) = ax + b|x| \quad ab = ?$$

سؤال ٦

$$f(x) = \begin{cases} rx & x > 0 \\ rx & x < 0 \end{cases} \xrightarrow{f^{-1}} \begin{cases} \frac{1}{r} x & x > 0 \\ \frac{1}{r} x & x < 0 \end{cases} \Rightarrow \begin{cases} a + b = \frac{1}{r} \\ a - b = \frac{1}{r} \end{cases}$$

$$ab = \frac{r}{\wedge} x - \frac{1}{\wedge} = -\frac{r}{4r}$$

$$\begin{aligned} ra &= \frac{r}{2} \Rightarrow a = \frac{\frac{r}{2}}{\frac{r}{2}} = \frac{r}{\wedge} \\ b &= -\frac{1}{\wedge} \end{aligned}$$

$$f(x) = \frac{rx + r}{x + m}$$

$$(r, -10) \Rightarrow -10 = \frac{4 + r}{r + m} \Rightarrow$$

سؤال ٧

$$\Rightarrow -r - m = 1 \Rightarrow \boxed{m = -r}$$

$$y = f \circ f^{-1}(x) + f^{-1}(x)$$

$$f^{-1}(x) \Rightarrow f^{-1}(x) = \frac{rx + r}{x - r}$$

$$f^{-1}(x) = f(x) \Leftrightarrow a = -d \text{ من } y = \frac{ax + b}{cx + d}$$

$$f(f^{-1}(x)) = x, y = x + \frac{rx + r}{x - r} \Rightarrow x + \frac{rx + r}{x - r} = x \Rightarrow rx + r = 0$$

$$\Rightarrow \boxed{x = -\frac{r}{r}}$$

$$f\left(\frac{r-rx}{a}\right) = r - g\left(\frac{x}{\frac{r}{r}}\right)$$

سؤال ٨

$$f g^{-1}(r) + f^{-1}(-r) = ?$$

$$g(b) = r \rightarrow g^{-1}(r) = b$$

$$f(a) = -r \rightarrow f^{-1}(-r) = a$$

$$rb + a \rightarrow f\left(\frac{x}{r}\right) - rx + r = f\left(\frac{x}{r}\right) - rx + r \Rightarrow rx - rx + r = r \Rightarrow \boxed{r}$$

$$y = \frac{2x - 13}{1 - x} \xrightarrow{\text{قرينة}} -y = \frac{-2x - 13}{1 + x} \Rightarrow y = \frac{2x + 13}{x + 1} \Rightarrow$$

سؤال ٩

$$y = \frac{2(x - r) + 13}{(x - r) + 1} = \frac{2x - 10 + 13}{x - 1} = \frac{2x + 3}{x - 1} - r \Rightarrow f(x) = \frac{rx + 4}{x - 1}$$

$$f^{-1}(x) = \frac{4 + x}{x - r}$$

$$\Rightarrow \frac{x + 4}{x - r} = \frac{rx + 4}{x - 1} \Rightarrow x^2 - rx - 4 = \dots \Rightarrow$$

$$\boxed{r = 3}$$

$$f(x) = x + f[x] \Rightarrow x = y + [fy] \Rightarrow y = x - [fy] \Rightarrow x = y - f[y] \text{ سؤال ١٠}$$

$$(fg)(x) = ? \Rightarrow [x] = [y - f[y]] \Rightarrow [x] = [y] + f[y] = 2[y]$$

$$[y] = \frac{[x]}{2} \rightarrow f[y] = \frac{f[x]}{2} \Rightarrow (f - g)(x) = f(x) - g(x) \rightarrow x + f[x] + \frac{f[x]}{2}$$

$$\rightarrow f[x] + \frac{f[x]}{2} \Rightarrow \frac{2f[x] + f[x]}{2} = \boxed{\frac{3f[x]}{2}}$$