Subject:

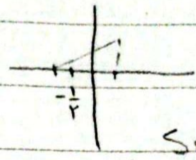
Year: Month: Day: ( ) دوازدهم ریاضی B تکلیف ۱۰ حدیث متین پور page: ( )

۱۶

۱)

$f: 2y = 3x - 1 \longrightarrow f^{-1}: 3x = 2y + 1 \quad y \overset{x}{\rightleftarrows} x = \dfrac{2y+1}{3}$

$\dfrac{2x+1}{3} = 0 \longrightarrow 2x + 1 = 0 \longrightarrow 2x = -1 \quad x = -\dfrac{1}{2}$

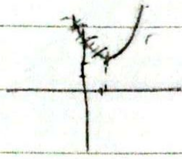


$S = \dfrac{\frac{1}{3} \times \frac{1}{2}}{2} = \dfrac{1}{12}$

۲

۲)

الف) $y = x^2 - 2x + 3 \quad x_s = 1, \; y_s = 2$



یک به یک هست $\quad D_{f^{-1}} = [2, +\infty)$

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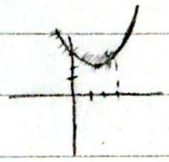
$y^{-1} = x^2 - 2x + 3 - 2 + 2$

$y = (x-1)^2 + 2 \longrightarrow \sqrt{y-2} = \sqrt{(x-1)^2} \longrightarrow |x-1| = \sqrt{y-2} \longrightarrow x - 1 = \sqrt{y-2}$

$y^{-1} = \sqrt{x-2} + 1$

ب) $y = x^2 - 2x + 3 \quad x_s = 1, \; y_s = 2$



یک به یک هست $\quad D_{f^{-1}} = [6, +\infty)$

$x = 3, \; y = 6$

$y^{-1} = \sqrt{x-2} + 1$

۳)

$y = 2x - \sqrt{16 - 4x(4-x)} \Longrightarrow 2x - \sqrt{4(x-2)^2} = 2x - 2|x-2|$

$16 - 16x + 4x^2 \qquad 2|x-2|$

$4(x^2 - 4x + 4)$

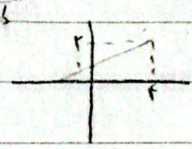
$(x-2)^2$

$2x + 2x - 4 \;|\; 2x - 2x + 4$

$4x - 4$

وارون پذیر

$y = 4x - 4$ وارون $4x = y + 4 \quad y \rightleftarrows x = \dfrac{y+4}{4}$



$S = \dfrac{4 \times (1+2)}{2} = 6$

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۴)

هم علامت

$y = \dfrac{x}{\sqrt{x^2 - 5}} \longrightarrow y^2 = \dfrac{x^2}{x^2 - 5} \longrightarrow y^2x^2 - 5y^2 = x^2$

$x^2(y^2 - 1) = 5y^2$

ارفاق

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$x_1 = \dfrac{x_1^2}{x_1^2 - 5} = \dfrac{x_2^2}{x_2^2 - 5} \Longrightarrow x_1^2 = x_2^2 \qquad x^2 = \dfrac{5y^2}{y^2 - 1} \longrightarrow y^2 = \dfrac{5x^2}{x^2-1} \rightarrow y = \pm\sqrt{\dfrac{5x^2}{x^2-1}}$

$x_1 = \pm x_2$

یک به یک هست ✓

حالت $\oplus$ قابل قبول است چون x و y هم علامت هستند

۵)

$$g(x) = \begin{cases} x^2 - 4x + k & x \geq 2 \xrightarrow{R} [k-4, +\infty) \\ -x^2 + 4x + 3k & x < 2 \xrightarrow{R} (-\infty, 3k+8) \end{cases}$$

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$k - 4 \geq 3k + 8$

$2k \leq -12 \rightarrow \boxed{k \leq -6}$

$y = 4x$

$x = 4y$

$\frac{x}{4} = y$

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$f(x) = 3x + |x| \begin{cases} \xrightarrow{x \geq 0} 4x \\ \xrightarrow{x < 0} 2x \end{cases} \longrightarrow f^{-1}(x) \begin{cases} \xrightarrow{x \geq 0} \frac{x}{4} \\ \xrightarrow{x < 0} \frac{x}{2} \end{cases}$

۲

$a = \frac{\frac{1}{4} + \frac{1}{2}}{2} = \frac{3}{8}$ $\quad x = 1, \; y = \frac{1}{4}$ $\quad a(1) + b = \frac{1}{4}$ $\quad b = \frac{1}{4} - \frac{3}{8} = -\frac{1}{8}$

$a \times b = \frac{3}{8} \times -\frac{1}{8} = \boxed{-\frac{3}{64}}$

(7

$f(x) = \frac{3x+4}{x+m} \xrightarrow[y=-10]{x=2} -10 = \frac{10}{2+m} \longrightarrow -20 - 10m = 10 \quad 10m = -30 \quad m = -3$

$f^{-1}(x) \Rightarrow \frac{3y+4}{y-3} = x \longrightarrow f^{-1} = \frac{-3x-4}{3-x}$

۲

$f^{-1}(x) + f \circ f^{-1}(x) = x \longrightarrow \frac{-3x-4}{3-x} + \cancel{x} = \cancel{x} \longrightarrow \frac{-3x-4}{3-x} = 0 \Rightarrow -3x - 4 = 0$

$-3x = 4$

$\boxed{x = -\frac{4}{3}}$

(8

$f(3-2x) = 2 - g\left(\frac{x}{2}\right) \longrightarrow \underbrace{g\left(\frac{x}{2}\right)}_{4} = 2 - f(3-2x) \rightarrow 2 - f(3-2x) = 4$

$f(3-2x) = -2$

$2 - g\left(\frac{x}{2}\right) = -2$ $\qquad f^{-1} \left| \begin{array}{l} -2 \\ 3-2x \end{array} \right.$

$g\left(\frac{x}{2}\right) = 4 \qquad g^{-1} \left| \begin{array}{l} 4 \\ \frac{x}{2} \end{array} \right.$

۲

$4 g^{-1}(4) + f^{-1}(-2) = 4 \times \frac{x}{2} + 3 - 2x =$

$2x + 3 - 2x = \boxed{3}$

(9

$y = \frac{5x-13}{1-x} \xrightarrow[y=-y]{x=-x} -y = \frac{-5x-13}{1+x} \xrightarrow{(x-2)} -y = \frac{-5(x-2)-13}{1+(x-2)} - 3$

$f = \frac{5x+3}{x-1} - 3 \longrightarrow \frac{5x+3-3x+3}{x-1} \rightarrow f = \frac{2x+6}{x-1} \longrightarrow f^{-1} \Rightarrow \frac{2y+6}{y-1} = x$

۲

$f^{-1} = \frac{x+6}{x-2} \qquad \frac{x+6}{x-2} = \frac{2x+6}{x-1} \rightarrow x^2 + 5x - 6 = 2x^2 + 2x - 12$

$x^2 - 3x - 6 \longrightarrow S = 3$

$[y] = [x] + 4[x] \rightarrow \frac{[y]}{5} = [x] \rightarrow f^{-1}(x) = x - 0.8[x]$

(10 ۰

$f(x) = x + 4[x] \longrightarrow -y = -x + 4[-x]$

$f \circ g = x + 4[x] - x + 0.8[x] = 4.8[x]$

PAYCO