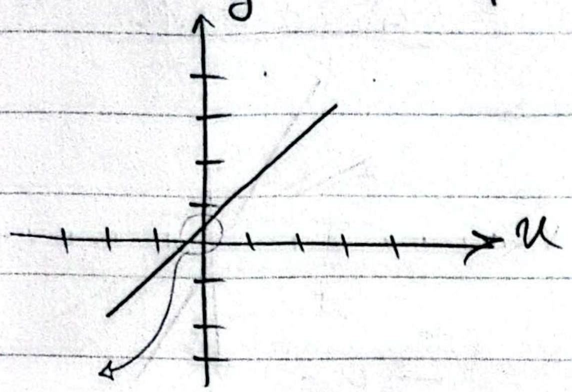
ترل رهنما

①

$2y = 3x - 1 \rightarrow y = \frac{3x-1}{2} \Rightarrow 2y = 3x - 1 \Rightarrow \frac{2y+1}{3} = x$



$y^{-1} = \frac{2x+1}{3}$

$(0, \frac{1}{3})\ (\frac{-1}{2}, 0)$

$\Delta\ \frac{1}{3},\ \frac{1}{2} \Rightarrow \frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{18} = S$

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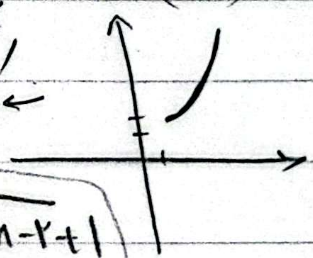
②

الف) $y = x^2 - 2x + 3 \quad x \in (1, +\infty) \Rightarrow \frac{-b}{2a} = \frac{2}{2} = 1 \quad (1, 2)$

یک به یک است



$x^2 - 2x + 3 \Rightarrow (x-1)^2 + 2 = y$

$\Rightarrow y - 2 = (x-1)^2 \Rightarrow \sqrt{y-2} = |x-1| \Rightarrow y^{-1} = \sqrt{x-2} + 1$

$R_y = D_{y^{-1}} \Rightarrow [2, +\infty)\ x \in$

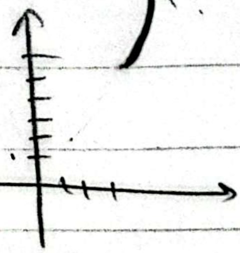
ب) $y = -2x + 3 + x^2 \quad x \in [3, +\infty) \Rightarrow x = 3 \Rightarrow 9 - 6 + 3 = 6$

$(x-1)^2 + 2 = y \Rightarrow \sqrt{y-2} = |x-1| \Rightarrow \sqrt{y-2} + 1 = x \Rightarrow \quad (3, 6)$

$y^{-1} = \sqrt{x-2} + 1$

$D_{y^{-1}} = R_y = [6, +\infty)\ x \in$

یک به یک است.



(۳)

$f(x) = -\sqrt{16-4x(4-x)} + 2x \Rightarrow 2x - \sqrt{16-16x+4x^2} \Rightarrow$

$2x - \sqrt{4(4-4x+x^2)} \Rightarrow 2x - 2\sqrt{(x-2)^2} \Rightarrow 2x - 2|x-2|$

$x>2$: $2x-2x+4=4$ → وارون پذیر نیست

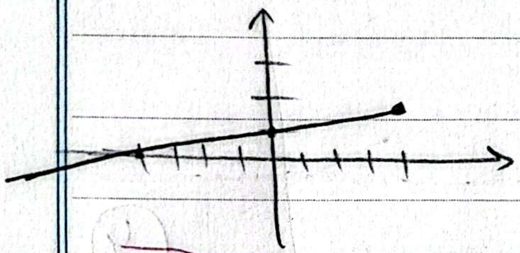
$x \leq 2$: $2x+2x-4 = 4x-4$

$y = 4x-4 \Rightarrow \frac{y+4}{4} = x \longrightarrow y^{-1} = \frac{x+4}{4} = \frac{x}{4}+1$

$x=2$: $8-4=4$

$R_y = (-\infty, 4]$

$R_y = D_{g^{-1}} \Rightarrow \frac{x}{4}+1 = y^{-1} \quad x \in (-\infty, 4]$



$S = 4 \times 1 \times \frac{1}{2} = 2$

(۴)

$y = \frac{x}{\sqrt{x^2-5}} \Rightarrow y_1 = y_2 \Rightarrow \frac{x_1}{\sqrt{x_1^2-5}} = \frac{x_2}{\sqrt{x_2^2-5}} \Rightarrow$

$\frac{x_1^2}{x_1^2-5} = \frac{x_2^2}{x_2^2-5} \Rightarrow x_2^2x_1^2 - 5x_2^2 = x_1^2x_2^2 - 5x_1^2 \Rightarrow$

$5x_2^2 = 5x_1^2 \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

$x_1 = x_2$ ← چون به توان ۲ رساندیم اشتباه شده

$y = \frac{x}{\sqrt{x^2-5}} \Rightarrow y\sqrt{x^2-5} = x \Rightarrow y^2x^2 - 5y^2 = x^2 \Rightarrow$

$y^2x^2 - x^2 = 5y^2 \Rightarrow \frac{5y^2}{y^2-1} = x^2 \Rightarrow x = \pm\sqrt{\frac{5y^2}{y^2-1}} \longrightarrow$

$x = \sqrt{\frac{5y^2}{y^2-1}}$ در صفحه نقاط تست می‌شود x و y هم علامت هستند

$D_{f^{-1}} = \mathbb{R} - [-1, 1]$

۵) از $\frac{-b}{2a}$ نتیجه میگیریم هر دو تابع به تنهایی وارون پذیرند.

$$g(x)=\begin{cases} x^2-4x+k & x\geqslant 2 \\ -x^2+6x+3k & x<2 \end{cases}$$

$$x^2-4x+k \Rightarrow \frac{-b}{2a}=\frac{4}{2}=2 \xRightarrow{x=2} 4-8+k=-4+k$$

$$-x^2+6x+3k \Rightarrow \frac{-b}{2a}=\frac{-6}{-2}=3 \xRightarrow{x=2} -4+12+3k=8+3k$$

$$-4+k\geqslant 8+3k \Rightarrow -12\geqslant 2k \Rightarrow -6\geqslant k$$

چون تابع اولی صعودی است
در دومی که نزولی شد اولی
باید حد بالای پایینی قرار بگیرد

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۶) $f(x)=3x+|x| \Rightarrow x>0 \Rightarrow 3x+x=4x=y$

$R_{f(x)}=(0,+\infty)$ $\quad f^{-1}_{(x)}=\frac{x}{4}$

$x\leqslant 0 \Rightarrow 3x-x=2x=y$ $\quad D_{f^{-1}}=[0,+\infty)$

$R_{f(x)}=(-\infty,0]$

$\frac{x}{2}=f^{-1}_{(x)} \Rightarrow D_{f^{-1}}=(-\infty,0]$

$$\begin{aligned} ax+bx&=\frac{x}{4} \\ ax-bx&=\frac{x}{2} \\ \hline 2ax&=\frac{3x}{4} \Rightarrow \end{aligned}$$

$a=\frac{3}{8}$

$b=\frac{-1}{8}$

$g(x)=\frac{3}{8}x-\frac{1}{8}|x|$

$a\times b=\frac{3}{8}\times\frac{-1}{8}=\frac{-3}{64}$

(۷)

$f(x)=\frac{3x+4}{x+m}$ $(2,-10)$ $\xrightarrow{x=2}$ $\frac{6+4}{2+m}=-10 \Rightarrow -20-10m+10$

$=30m-10m \Rightarrow m=-3$

$f(x)=\frac{3x+4}{x-3} \Rightarrow \underbrace{f\circ f^{-1}(x)}+f^{-1}(x)$

$x+\frac{3x+4}{x-3}=x \Rightarrow \frac{3x+4}{x-3}=0$

$y^{-1}=-\frac{-3x-4}{x-3}=y^{-1}=\frac{3x+4}{x-3}$

$x=-\frac{4}{3}$

(۸)

$f(3-2x)=2-g(\frac{x}{2})$

$g^{-1}(4)=t \rightarrow g(t)=4 \rightarrow \frac{x}{2}=t \Rightarrow x=2t$

$f(3-4t)=2-g(\frac{2t}{2}) \rightarrow f(3-4t)=-2 \rightarrow f^{-1}(-2)=$

$3-4t$

$4t+3-4t=3$

(۹)

$y=\frac{5x-13}{1-x} \rightarrow -f(-x) \Rightarrow y=\frac{5x+13}{x+1} \rightarrow f(x-2)-3$

$\rightarrow \frac{5(x-2)+13}{(x-2)+1}-3 \Rightarrow \frac{5x-10+13}{x-1}-3 \Rightarrow \frac{5x+3-3x+3}{x-1}$

$\Rightarrow \frac{2x+6}{x-1} \rightarrow f^{-1}(x)=\frac{-(-x-6)}{x-2}=\frac{x+6}{x-2} \Rightarrow$

$\frac{x+6}{x-2}=\frac{2x+6}{x-1} \rightarrow x^2+5x-6=2x^2-4x+6x-12 \Rightarrow$

$x^2-3x-6=0 \xrightarrow{\text{مجموع طول نقاط}} +3$

$$f(x) = x + 2[x] \Rightarrow [y] = [x + 2[x]] = [y] = [x] + 4[x] \quad (b)$$

$$\Rightarrow [y] = 5[x] \Longrightarrow \frac{[y]}{5} = [x] \xrightarrow{[x] = \frac{y-x}{4}} \frac{[y]}{5} = \frac{y-x}{4}$$

$$\Longrightarrow \frac{4[y]}{5} - y = -x \Rightarrow y^{-1} = x - \frac{4[x]}{5}$$

$R = R$

$D_f = R$

$$x + 4[x] - x + \frac{4[x]}{5} \Rightarrow \frac{20[x] + 4[x]}{5} = \frac{24[x]}{5}$$