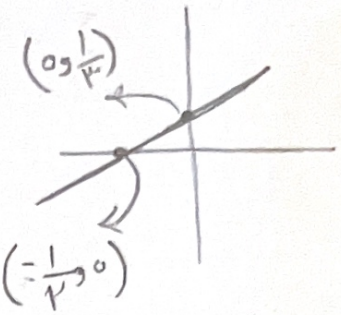
لسا راعبدالهی :

۱)

$2y = 3x - 1$

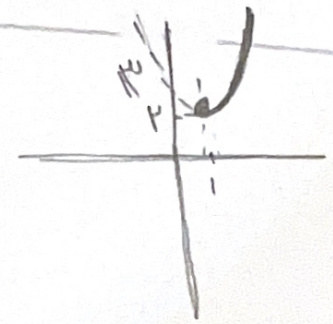
$y = \frac{3x-1}{2}$ $\quad f^{-1}(x) = \frac{3}{2}x + \frac{1}{2}$



$S = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{12}$

---

۲)



الف) $y = x^2 - 2x + 2$

$(x-1)^2 + 1$

☑ یک به یک

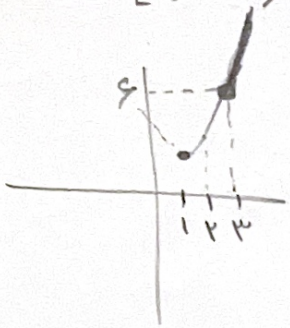
$x = (y-1)^2 + 1$

$1 + \sqrt{x-1} = y$ $\quad x \in [1, +\infty)$

ب) $y = x^2 - 2x + 2$

☑ یک به یک

$f^{-1}(x) = 1 + \sqrt{x-1}$ $\quad x \in [1, +\infty)$



$2x - \sqrt{16 - 16x + 4x^2}$

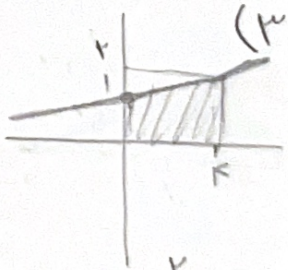
$2x - 2\sqrt{x^2 - 4x + 4} = 2x - 2|x-2|$

$x \geq 2$ $\quad 2x - 2x + 4 = 4$ X

$x \leq 2$ $\quad 2x + 2x - 4 = 4x - 4$ $\checkmark$ در بازه $(-\infty, 2]$

$f^{-1}(x) = \frac{x+4}{4} = \frac{x}{4} + 1$ $\quad D_{f^{-1}} = (-\infty, 4]$

۳)



$S = \frac{(1+2) \times 4}{2} = 6$

(۴

$y_1 = \frac{x_1}{\sqrt{x_1^2 - 5}}$ $\quad y_2 = \frac{x_2}{\sqrt{x_2^2 - 5}}$ $\quad y_1 = y_2 \xrightarrow{:} x_1 = x_2$

$\frac{x_1}{\sqrt{x_1^2-5}} = \frac{x_2}{\sqrt{x_2^2-5}} \Rightarrow \frac{x_1^2}{x_1^2-5} = \frac{x_2^2}{x_2^2-5} \Rightarrow \cancel{x_1^2 x_2^2} - 5x_2^2 = \cancel{x_1^2 x_2^2} - 5x_1^2$

$x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2 \xrightarrow[\text{هم علامت}]{x, y} x_1 = x_2$

$x = \frac{y}{\sqrt{y^2-5}} \Rightarrow x^2 = \frac{y^2}{y^2-5}$ $\quad x^2y^2 - 5x^2 = y^2$

$-5x^2 = y^2(1-x^2)$

$y = \sqrt{\frac{-5x^2}{1-x^2}}$

(۵

به ازای $x=2$

$4 - 8 + K = -4 + K$

$-4 + 12 + 3K = 8 + 3K$

شرط یکبه‌یک بودن:

$8 + 3K \leq -4 + K$

$2K \leq -12$

$K \leq -6$

(۶

$x \geq 0 \quad 3x + x = 4x$

$x < 0 \quad 3x - x = 2x$

$f^{-1}(x) \quad x \geq 0 \quad \frac{1}{4}x$

$x < 0 \quad \frac{1}{2}x$

$\frac{\frac{1}{4} + \frac{1}{2}}{2} = a = \frac{3}{8}$ $\quad f(1) = 4 \to f^{-1}(4) = 1$

$g(4) = \frac{3}{\cancel{8}} \times \cancel{4} + 4b = 1$

$4b = -\frac{1}{2} \quad b = -\frac{1}{8}$ $\quad ab = \frac{3}{8} \times (-\frac{1}{8}) = -\frac{3}{64}$

سارا عبدالحي

$$\frac{3(2)+k}{2+m} = -10 \qquad m = -3$$

-٧

$$x = \frac{3y+k}{y-3} \qquad yx - 3x = 3y + k \qquad y = \frac{3x+k}{x-3}$$
$$-3x - k = y(3-x)$$

$$f\left(\frac{3x+k}{x-3}\right) + \frac{3x+k}{x-3}$$

$$\frac{3\left(\frac{3x+k}{x-3}\right)+k}{\frac{3x+k}{x-3}-3} = \frac{\frac{9x+12+kx-12}{x-3}}{\frac{3x+k-3x+9}{x-3}} = \frac{12x}{12} = x$$

$$(f \circ f^{-1}(x) = x)^{*} \Rightarrow x + \frac{3x+k}{x-3} = \frac{x^2+k}{x-3} \Rightarrow \frac{x^2+k}{x-3} = x$$

$$x^2 + k = x^2 - 3x \qquad \boxed{x = -\frac{k}{3}}$$

-٨

$$f^{-1}(-2) = a$$

$$f(a) = -2$$

$$3 - 2x = a$$

$$x = \frac{a-3}{-2} = \frac{3-a}{2}$$

$$f(a) = 2 - g\left(\frac{3-a}{k}\right)$$

$$g\left(\frac{3-a}{k}\right) = k$$

$$g^{-1}(k) = \frac{3-a}{k}$$

$$kg^{-1}(k) + f^{-1}(-2) = k\left(\frac{3-a}{k}\right) + a = 3$$

(۹

$$y = \frac{+5x + 11}{1+x}$$

قرینه

$$\longrightarrow \frac{5(x-2)+11}{1+(x-2)} - 3 = f(x) = \frac{2x+4}{x-1}$$

$$\text{وارون} \longrightarrow x = \frac{2y+4}{y-1} \Rightarrow xy - x = 2y + 4 \Rightarrow -x = y(2-x) + 4$$

$$\frac{-x-4}{2-x} = y \qquad y = \frac{x+4}{x-2}$$

$$\frac{x+4}{x-2} = \frac{2x+4}{x-1} \qquad x^2 + 5x - 4 = 2x^2 + 2x - 12$$

$$x^2 - 3x - 4 = 0$$

$$-\frac{b}{a} = S = 3$$

(۱۰

$$y = x + f[x]$$

$$x = y + f[y] \xrightarrow{\text{برابر}} [x] = [y] + f[y] \rightarrow [y] = \frac{[x]}{5}$$

$$y = -f[y] + x$$

$$y = x - \frac{f[x]}{5} = g(x)$$

$$f(x) - g(x) = \cancel{x} + f[x] - \cancel{x} + \frac{f[x]}{5} = \frac{14f[x]}{5}$$