

دوازدهم حضرت B

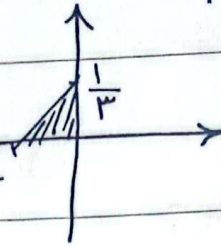
تکلیف 10

کمیای امیری بنا

$$r_y = 3x - 1 \xrightarrow{\text{دارد}} r_x = 3y - 1 \rightarrow r_{x+1} = 3y \rightarrow y = \frac{r_{x+1}}{3} \quad (1)$$

$$x=0 \rightarrow y = \frac{1}{3} \quad \left| \frac{1}{3} \right.$$

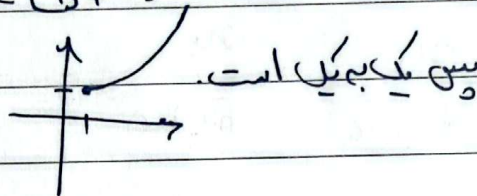
$$y=0 \rightarrow \frac{r_{x+1}}{3} = 0 \rightarrow r_x = -1 \quad \left| \frac{-1}{3} \right. \quad \frac{-1}{3}$$



$$S = \frac{1}{3} \times \frac{1}{3} = \frac{1}{12}$$

$$f(x) = x^2 - 2x + 3 \quad x \in [1, +\infty) \quad (2)$$

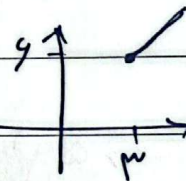
$$-\frac{b}{2a} = \frac{2}{2} = 1$$



$$y = x^2 - 2x + 3 + 1 - 1 \rightarrow y = (x-2)^2 - 1 \xrightarrow{\text{دارد}}$$

$$x = (y-2)^2 - 1 \rightarrow (y-2)^2 = x+1 \rightarrow y-2 = \pm\sqrt{x+1} \rightarrow$$

$$y-2 = +\sqrt{x+1} \rightarrow y = \sqrt{x+1} + 2 \quad D_{f^{-1}} = R_f = [1, +\infty)$$

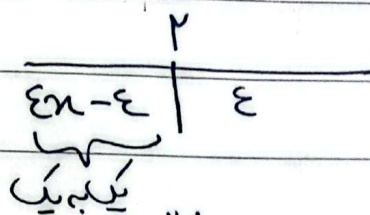


$$f(x) = x^2 - 2x + 3 \quad x \in [1, +\infty)$$

$$y = x^2 - 2x + 3 + 1 - 1 \xrightarrow[\text{اف}]{\text{دارد}} y = \sqrt{x+1} + 2 \quad D_{f^{-1}} = R_f = [1, +\infty)$$

$$f(x) = \sqrt{14 - 4x(x-1)} = \sqrt{14 - 4x^2 + 4x} \quad (3)$$

$$= \sqrt{14 - 4(x-1)^2} = \sqrt{14 - 4(x-1)^2} \quad D_f = R_f = x \leq 1$$



$$y = \epsilon x - \epsilon \quad x \leq 1 \quad \text{واری} \rightarrow x = \frac{y + \epsilon}{\epsilon}$$

$$\epsilon y = x + \epsilon \rightarrow y = \frac{x}{\epsilon} + 1 \quad \begin{matrix} x=0 \rightarrow y=1 \\ x=1 \rightarrow y=2 \end{matrix}$$

$$S = \frac{1}{2} \times 1 + \frac{1}{2} \times 1 = 1 \quad (4)$$

$$y = \frac{x}{\sqrt{x^2 - a}}$$

(5)

$$\begin{matrix} (x_1, y) \\ (x_2, y) \end{matrix} \rightarrow y = \frac{x_1}{\sqrt{x_1^2 - a}} = \frac{x_2}{\sqrt{x_2^2 - a}} \quad \text{توالت} \rightarrow \frac{x_1^2}{x_1^2 - a} = \frac{x_2^2}{x_2^2 - a}$$

$$\rightarrow x_1^2 x_2^2 - a x_1^2 = x_1^2 x_2^2 - a x_2^2 \rightarrow x_1^2 = x_2^2 \rightarrow x_1 = \pm x_2$$

اما چون  $x_1$  و  $x_2$  هم علامت دارند و  $x_1 = x_2$  فقط، پس  $x_1 = x_2$  است.

$$y = \frac{x}{\sqrt{x^2 - a}} \quad \text{توالت} \rightarrow y(\sqrt{x^2 - a}) = x \quad \text{توالت} \rightarrow$$

$$y^2(x^2 - a) = x^2$$

$$y^2 x^2 - a y^2 = x^2 \rightarrow a y^2 = x^2 (y^2 - 1) \rightarrow x^2 = \frac{a y^2}{y^2 - 1}$$

$$\rightarrow x = \pm \frac{y \sqrt{a}}{\sqrt{y^2 - 1}} \quad \text{توالت} \rightarrow x = + \frac{y \sqrt{a}}{\sqrt{y^2 - 1}} \rightarrow f^{-1} = y = \frac{x \sqrt{a}}{\sqrt{x^2 - a}}$$

$$g(x) = \begin{cases} x^2 - \varepsilon x + k & x \geq 1 \\ -x^2 + 4x + 3k & x < 1 \end{cases}$$

(5)

$$x=1 \rightarrow \begin{aligned} & x^2 - \varepsilon x + k = \varepsilon - \varepsilon + k = k - \varepsilon \\ & -x^2 + 4x + 3k = -\varepsilon + 4 + 3k = 3k + 4 - \varepsilon \end{aligned}$$

$$3k + 4 - \varepsilon \leq k - \varepsilon \rightarrow 2k \leq -4 \rightarrow \boxed{k \leq -2}$$

$$f(x) = 3x + |x|$$

(6)

$$\begin{array}{c|c} 0 & \\ \hline 3x & \varepsilon x \end{array} \rightarrow f^{-1}(x) = \begin{cases} \frac{x}{\varepsilon} & x \geq 0 \\ \frac{x}{3} & x < 0 \end{cases}$$

$$R = (-\infty, 0) \quad R = [0, +\infty)$$

$$y = 3x \xrightarrow{a,b} y = \frac{x}{\varepsilon} \quad y = \varepsilon x \xrightarrow{a,b} y = \frac{x}{\varepsilon}$$

$$\begin{aligned} x \geq 0 & \\ (a+b)x &= \frac{x}{\varepsilon} \rightarrow (a+b) = \frac{1}{\varepsilon} \\ x < 0 & \\ (a-b)x &= \frac{x}{\varepsilon} \rightarrow (a-b) = \frac{1}{\varepsilon} \end{aligned} \left\{ \begin{aligned} & a = \frac{1}{\varepsilon} \rightarrow a = \frac{1}{\varepsilon}, b = \frac{1}{\varepsilon} \\ & a = \frac{1}{\varepsilon} \rightarrow a = \frac{1}{\varepsilon}, b = \frac{1}{\varepsilon} \end{aligned} \right.$$

$$\boxed{ab = \frac{1}{\varepsilon^2}}$$

$$f(n) = \frac{3n+4}{n+m} \quad \frac{(2, -1)}{2+m} \quad \frac{4+4}{2+m} = -1 \quad (V)$$

$$\frac{1}{2+m} = -1 \rightarrow 2+m = -1 \rightarrow \boxed{m = -3}$$

$$f(n) = \frac{3n+4}{n-3}$$

دو دو صفت این دو تابع برابر  
 ۰۰ و ۰۰ مکتوب تابع  
 همدتا تابع  $\rightarrow f^{-1} = f$

$$f \circ f^{-1}(n) = n \rightarrow y = n + f(n) = \frac{3n+4}{n-3} + n = n$$

$f^{-1}(n) = f(n)$  بنیم ساز تابع اول دوم

$$\frac{3n+4}{n-3} = 0 \rightarrow \boxed{n = -\frac{4}{3}}$$

$$f(3-2n) = 2 - g\left(\frac{n}{2}\right)$$

$$g^{-1}(4) + f^{-1}(-2) = ? \quad (A)$$

$$g^{-1}(4) = t \rightarrow g(t) = 4 \rightarrow \frac{n}{2} = t \rightarrow n = 2t$$

$$f(3-2(2t)) = 2 - g(t) = 2 - 4 = -2 \rightarrow f(3-4t) = -2$$

$$g^{-1}(4) + f^{-1}(-2) = \cancel{4} + 3 - \cancel{4} = \boxed{3}$$

$$y = \frac{5x-13}{1-x} \xrightarrow{\text{مضرب}} y = -\frac{-5x-13}{x+1} = \frac{5x+13}{x+1} \quad (9)$$

$$\begin{aligned} &\xrightarrow{\text{۲ واحد ۲}} = \frac{5(n-2)+13}{(n-2)+1} - 3 = \frac{2n+4}{n-1} \\ &\xrightarrow{\text{۳ واحد ۳}} \end{aligned}$$

$$\xrightarrow{\text{وادی}} n = \frac{2y+4}{y-1} \rightarrow y = \frac{\frac{2y+4}{y-1}+4}{\frac{2y+4}{y-1}-2}$$

$$f(n) = f^{-1}(n) \rightarrow \frac{2n+4}{n-1} = \frac{n+4}{n-2} \rightarrow 2n^2-5n+4n-2 = n^2+5n-4$$

$$\rightarrow n^2-3n-4=0 \rightarrow S = \frac{-b}{a} = \frac{3}{1} = 3$$

$$f(n) = n + \varepsilon[n] \quad g(n) = f^{-1}(n) \quad (10)$$

$$y = n + \varepsilon[n] \xrightarrow{\text{برای } n \in \mathbb{Z}} [y] = [n + \underbrace{\varepsilon[n]}_{\in \mathbb{Z}}] \rightarrow [y] = [n] + \varepsilon[n]$$

$$\rightarrow [y] = 5[n] \rightarrow [n] = \frac{[y]}{5}$$

$$y = n + \varepsilon \frac{[y]}{5} \rightarrow n = y - \varepsilon \frac{[y]}{5} \xrightarrow{\text{وادی}} y = n - \varepsilon \frac{[n]}{5}$$

$$f - g = n + \varepsilon[n] - n + \varepsilon \frac{[n]}{5} = \frac{4\varepsilon[n]}{5}$$