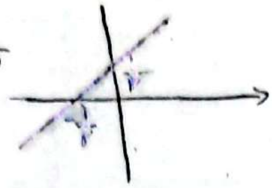
صبا وفازاده تکلیف شماره ۱۰ دوازدهم دختر B

۱) $2x = 3y - 1 \rightarrow y = \frac{2}{3}x + \frac{1}{3}$

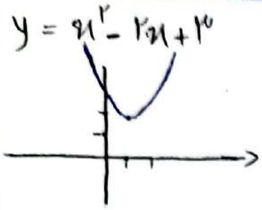
$x = 0 \rightarrow y = \frac{1}{3}$

$y = 0 \rightarrow x = -\frac{1}{2}$



$S = \frac{\frac{1}{3} \times \frac{1}{2}}{2} = \frac{1}{12}$

۲) $y = x^2 - 2x + 3$



$\frac{-b}{2a} = 1$ یک به یک هستند چون طول راس در دامنه آنها نیست

$y = x^2 - 2x + 1 + 2$ $\quad y = (x-1)^2 + 2$

$x = (y-1)^2 + 2 \rightarrow y = \sqrt{x-2} + 1$ $\quad f^{-1} = \sqrt{x-2} + 1$

$R_f = D_{f^{-1}}$

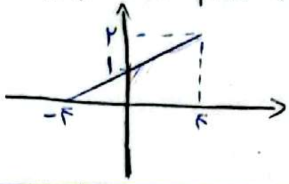
الف) $f^{-1} = \sqrt{x-2} + 1 \; ; \; x \geq 2$

ب) $f^{-1} = \sqrt{x-2} + 1 \; ; \; x \geq 6$

۳) $f(x) = 2x - \sqrt{16 - 16x + 4x^2}$ $\quad f(x) = 2x - \sqrt{(2x-4)^2} \rightarrow f(x) = 2x - |2x - 4|$

	2	
	$4x-4$	4

تابع f در بازه $x \geq 2$ وارون پذیر است $\quad f^{-1} = \frac{x+4}{4} \; ; \; x \leq 4$



$S = \frac{8 \times 2}{2} = 8$

۴) $y_1 = y_2$ $\quad \frac{x_1}{\sqrt{x_1^2 - 5}} = \frac{x_2}{\sqrt{x_2^2 - 5}}$ $\quad \frac{x_1^2}{x_1^2 - 5} = \frac{x_2^2}{x_2^2 - 5}$ $\quad x_1^2 x_2^2 - 5x_1^2 = x_1^2 x_2^2 - 5x_2^2 \rightarrow x_1^2 = x_2^2$

$x_1 = \pm x_2$ یک y نمی‌تواند دو x مختلف العلامت داشته باشد $\rightarrow x_1 = x_2$ $\quad x = \frac{y}{\sqrt{y^2 - 5}}$ $\quad x^2 = \frac{y^2}{y^2 - 5}$ $\quad x^2 y^2 - 5x^2 = y^2$

$y^2(x^2 - 1) = 5x^2$ $\quad y^2 = \frac{5x^2}{x^2 - 1}$ $\quad y = \pm \frac{\sqrt{5}x}{\sqrt{x^2 - 1}}$ x و y هم علامت $\rightarrow f^{-1} = \frac{\sqrt{5}x}{\sqrt{x^2 - 1}}$

۵) $x^2 - 4x + K : \frac{-b}{2a} = 2$ $\quad -x^2 + 6x + 3K : \frac{-b}{2a} = 3$ $\quad x = 2 \rightarrow x^2 - 4x + K = K - 4$

$x = 2 \rightarrow -x^2 + 6x + 3K = 8 + 3K$

$8 + 3K \leq K - 4$ $\quad K \leq -6$

۶) $f^{-1} = \begin{cases} \frac{1}{4}x & x > 0 \\ \frac{1}{2}x & x < 0 \end{cases}$ $\quad a = \frac{\frac{1}{4} + \frac{1}{2}}{2} = \frac{3}{8}$ $\quad b = \frac{\frac{1}{4} - \frac{1}{2}}{2} = -\frac{1}{8}$ $\quad ab = \frac{3}{8}(-\frac{1}{8}) = -\frac{3}{64}$

۷) $-10 = \frac{6+4}{2+m}$ $\quad 2 + m = -1$ $\quad m = -3$ $\quad f(x) = \frac{3x+4}{x-3} \xrightarrow{a=-d} f(x) = f^{-1}(x)$ $\quad fof^{-1}(x) = x$

$x + \frac{3x+4}{x-3} = x \rightarrow x = \frac{-4}{3}$

۸) $f^{-1}(2 - g(\frac{x}{2})) = 3 - 2x$ $\quad 2 - g(\frac{x}{2}) = -2$ $\quad g(\frac{x}{2}) = 4 \rightarrow g^{-1}(4) = \frac{x}{2}$ $\quad f^{-1}(-2) = 3 - 2x$

$4g^{-1}(4) + f^{-1}(-2) = 4(\frac{x}{2}) + 3 - 2x = 2x + 3 - 2x = 3$

۹) $y = \frac{5x - 13}{1 - x} \xrightarrow{\text{قرینه نسبت به مبدا}} -y = \frac{-5x - 13}{1 + x}$ $\quad y = \frac{5x + 13}{1 + x}$ $\quad f(x) = \frac{5(x-2) + 13}{1 + (x-2)} - 3$ $\quad f(x) = \frac{2x+6}{x-1}$

$x = \frac{2y + 6}{y - 1}$ $\quad y = -\frac{-x - 6}{x - 2}$ $\quad f^{-1}(x) = \frac{x+6}{x-2}$ $\quad \frac{2x+6}{x-1} = \frac{x+6}{x-2}$ $\quad 2x^2 + 2x - 12 = x^2 + 5x - 6$

$x^2 - 3x - 6 = 0$ $\quad$ جمع ریشه‌ها $= 3$

۱۰) $y = x + 4[x]$ $\quad x = y + 4[y]$ $\quad y = x - 4[y]$ $\quad [x] = [y + 4[y]]$ $\quad [x] = [y] + 4[y]$

$[x] = 5[y]$ $\quad [y] = \frac{[x]}{5}$ $\quad g(x) = x - \frac{4[x]}{5}$ $\quad (f-g)(x) = x + 4[x] - x + \frac{4[x]}{5} = \frac{24}{5}[x]$