رئیسی تبریزی

۱) $x=0 \to y=-\frac{1}{2}$, $y=0 \to x=\frac{1}{3}$ $\Rightarrow f^{-1} \to x=0 \to y=\frac{1}{3}$, $y=0 \to x=-\frac{1}{2}$ $\Rightarrow S = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{12}$ $\to S_f = S_{f^{-1}}$ (طول از مبدا و عرض از مبدا در نتیجه حاصل ضرب یکسان.)

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۲)

الف] $x^2-2x+1+2 \to (x-1)^2+2 \to -\frac{b}{2a}=\text{①} \to [1,+\infty) \to$ یک به یک $\to y=(x-1)^2+2 \to y-2=(x-1)^2$

$\Rightarrow y = \pm\sqrt{y-2}+1 \Rightarrow \boxed{y=\sqrt{x-2}+1} \Rightarrow R_{f^{-1}} = R_f \Rightarrow x-2 \geq 0 \to x \geq 2 \to \boxed{1-2+3=\text{②} \to [2,+\infty)}$

(−) ق ق ، $x \geq 1$

ب] $\Rightarrow -\frac{b}{2a}=\text{①} \to [3,+\infty) \xrightarrow{\text{یک به یک}} \boxed{y=\sqrt{x-2}+1} \Rightarrow R_{f^{-1}} = R_f \Rightarrow 3 \Rightarrow 9-2(3)+3=\text{⑥}$

$\to [6,+\infty)$

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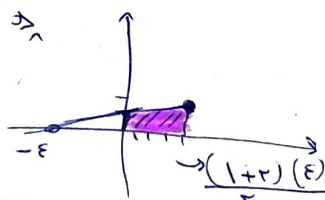
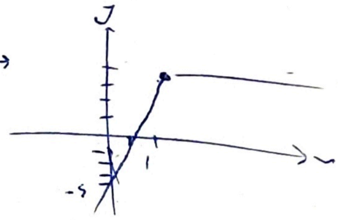
۳) $f(x)=2x-\sqrt{16-16x+4x^2} = 2x-\sqrt{4(x^2-4x+4)} = 2x-\sqrt{4(x-2)^2} \Rightarrow 2x-2|x-2|$

$x>2 \Rightarrow 2x-2(x-2) \to 2x-2x+4=\text{④}$ / $x<2 \Rightarrow 2x+2x-4 \Rightarrow 4x-4) \to$

$\Rightarrow (-\infty,1] \Rightarrow y=4x-4 \Rightarrow y+4=4x \to \frac{x}{4}+1=y \Rightarrow$

$D_f \to R_f = (-\infty,4]$





$\to \frac{(1+2)(4)}{2}=3(2)=\text{⑥}$

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۴) $y=\frac{x}{\sqrt{x^2-5}} \Rightarrow y_1=y_2 \to \frac{x_1}{\sqrt{x_1^2-5}}=\frac{x_2}{\sqrt{x_2^2-5}} \Rightarrow x_1^2(x_2^2-5)=x_2^2(x_1^2-5) \Rightarrow x_1^2x_2^2-5x_1^2=x_2^2x_1^2-5x_2^2$

$x_1^2=x_2^2 \to x_1=\pm x_2$ } ⇒ (غیر ممکن) ⇒ $y=\frac{x}{\sqrt{x^2-5}}$ → هم علامت هستند x و y | $\to x \neq -1$ ⟶ یک به یک است.

$y\sqrt{x^2-5}=x \to y^2(x^2-5)=x^2 \to y^2x^2-5y^2=x^2 \to 5y^2=y^2x^2-x^2 \to x^2(y^2-1)=5y^2 \Rightarrow x^2=\frac{5y^2}{y^2-1} \Rightarrow x=\pm\sqrt{\frac{5y^2}{y^2-1}}$

$\boxed{x=\sqrt{\frac{5y^2}{y^2-1}}} \longrightarrow \boxed{y=\sqrt{\frac{5x^2}{x^2-1}}}$

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۵) ~~...~~ $\Rightarrow \text{②} \to 4-8+k \geq -4+12+3k \to k-4 \geq 8+3k$

$-12 \geq 2k \to -6 \geq k$

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۶)

$x>0 \to 3x+x=4x \to y=4x \to x=\frac{1}{4}y \to y=\frac{x}{4}$

$x<0 \to 2x \Rightarrow y=\frac{x}{2}$

$x>0$	$\frac{x}{4}$
$x<0$	$\frac{x}{2}$

$ax+b|x| \Rightarrow ax+bx$

$(a+b)x=\frac{x}{4}$

$(a-b)x=\frac{x}{2}$

$a+b=\frac{1}{4}$, $a-b=\frac{1}{2}$ $\Rightarrow 2a=\frac{3}{4} \Rightarrow \boxed{a=\frac{3}{8}}$

$\boxed{b=\frac{2}{8}-\frac{3}{8}=-\frac{1}{8}}$ $\Longrightarrow \frac{3}{8}\times-\frac{1}{8}=-\frac{3}{64}$

۷) $\frac{6+4}{2+m} = -10 \Rightarrow 2+m=-1 \Rightarrow m=-3$ $\Rightarrow f \circ f^{-1}(x) \Rightarrow (x) \rightarrow \cancel{x} + f^{-1}(x) = \cancel{x} \Rightarrow f^{-1}(x) = 0$

$\frac{3x+4}{x-3} \Rightarrow (0) \Rightarrow \frac{4}{-3} = -\frac{4}{3}$

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۸)

$\frac{g}{}$

~~$f^{-1}(-2) = g(-1)$~~

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۸) $f(3-2x) = t \xrightarrow{f^{-1}} 3-2x = f^{-1}(t) \Rightarrow x = \frac{3-f^{-1}(t)}{2}$ / $g(\frac{x}{2}) = 2-t \Rightarrow \frac{x}{2} = g^{-1}(2-t) \Rightarrow x = 2g^{-1}(2-t)$

$\Rightarrow \frac{3-f^{-1}(t)}{2} = 2g^{-1}(2-t) \Rightarrow t=-2 \Rightarrow 3-f^{-1}(-2) = 4g^{-1}(4) \Rightarrow (3)$

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۹) $-\left(\frac{-5x-13}{1+x}\right) \Rightarrow \frac{5x+13}{x+1} \Rightarrow \frac{5(x-2)+13}{x-2+1} - 3 \Rightarrow \frac{5x+3}{x-1} - 3 = \frac{2x+6}{x-1}$

$\frac{2x+6}{x-1} = x \rightarrow x^2 - x = 2x+6 \rightarrow x^2-3x-6=0 \rightarrow -\frac{b}{a} = 3$

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۱۰)

$y = x + 4[x] \Rightarrow f^{-1} = g(x) \Rightarrow f-g \Rightarrow f-f^{-1}(x))$

$[y] = [x+4[x]] \Rightarrow 5[x] = [y] \Rightarrow y = x+4[x] \Rightarrow [x] = \frac{[y]}{5} \Rightarrow y = x + \frac{4[y]}{5}$

$\frac{y-4[y]}{5} = x \Rightarrow f^{-1} \rightarrow y = \frac{5x-4[x]}{5} \Rightarrow f-g \Rightarrow x+4[x] - \frac{5x-4[x]}{5} \Rightarrow 4[x] + \frac{4[x]}{5} \Rightarrow \frac{24}{5}[x] =$

$4/8[x]$