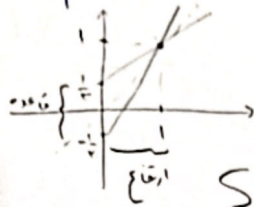


کارانه پوری

تکلیف ۵۱

کلاس دوازدهم

۱) $y = \frac{2x-1}{x}$
 $y' = \frac{2x+1}{x^2} \rightarrow \frac{2x+1}{x^2} = 0$



$\frac{2x-1}{x} = 2 \rightarrow x = 1$
 $S = \frac{(\frac{1}{2} + \frac{1}{2}) \times 1}{2} = \boxed{\frac{1}{2}}$ جواب

۲)

۱) $f(x) = x^2 - 2x + 2 \rightarrow \frac{b}{2a} = 1$
 $x = 1 \rightarrow y = 1$

$y = (x-1)^2 + 1 \rightarrow \sqrt{x-2} + 1, y$

$f^{-1}(x) = \sqrt{x-2} + 1$

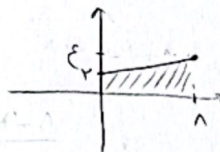
۲) $g(x) = x^2 - 2x + 2 \rightarrow \frac{b}{2a} = 1$
 $x = 3 \rightarrow y = 4$

$g^{-1}(x) = \sqrt{x-2} + 1, x \in [4, +\infty)$

۳)

$f(x) = 2x - \sqrt{x(x-1)} \Rightarrow 2x - 2|x-1| \rightarrow 2x - 2|x-1|$

$2x+2x-1 = 4x-1 = 2x \Rightarrow x = \frac{1}{2}$
 $2x-1 = 0 \Rightarrow x = \frac{1}{2}$
 $f^{-1}(x) = \frac{x+1}{x} \rightarrow \frac{1}{x}$



$S = \frac{(x+1)x}{2} = \boxed{2}$ جواب

۴)

$y = \frac{x}{\sqrt{x^2-1}} \Rightarrow y_1 = \frac{x_1}{\sqrt{x_1^2-1}}, y_2 = \frac{x_2}{\sqrt{x_2^2-1}} \Rightarrow \frac{y_1}{\sqrt{y_1^2-1}} = \frac{y_2}{\sqrt{y_2^2-1}}$

$\frac{x_1^2}{x_1^2-1} = \frac{x_2^2}{x_2^2-1} \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

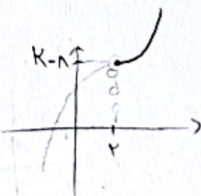
$y = \frac{x}{\sqrt{x^2-1}} \Rightarrow x = \frac{y}{\sqrt{y^2-1}} \Rightarrow x^2 = \frac{y^2}{y^2-1} \Rightarrow x^2(y^2-1) = y^2 \Rightarrow -x^2 = (1-x^2)y^2 \Rightarrow \frac{-x^2}{1-x^2} = y^2$

$f^{-1}(x) = \frac{\sqrt{\omega x^2}}{\sqrt{x^2-1}} \Rightarrow \frac{x\sqrt{\omega}}{\sqrt{x^2-1}}$

۵)

$\frac{b}{2a} = 1$

$g(x) = \begin{cases} x^2 - 2x + 2; x \geq 1 \rightarrow \frac{2}{x-1} \\ -x^2 + 4x + 2; x < 1 \rightarrow \frac{2}{x+1} \end{cases}$



$xK+1 \leq K-1 \Rightarrow xK \leq -2 \Rightarrow K \leq -1$
 $K \in (-\infty, -1]$
 $K \leq -1$ جواب

④ $y = \frac{1}{x} + |x|$ $f^{-1}(x) = \begin{cases} \frac{x}{2} & x \leq 0 \\ \frac{x}{2} & x \geq 0 \end{cases}$ $\frac{1}{x} + \frac{1}{x} = \frac{2}{x} \Rightarrow \frac{x}{2} = \frac{1}{x} \Rightarrow x^2 = 2 \Rightarrow x = \pm \sqrt{2}$ $\frac{1}{x} - \frac{1}{x} = 0 \Rightarrow \frac{x}{2} = 0 \Rightarrow x = 0$ $\boxed{ab = \frac{r}{4\epsilon}}$ جواب

⑤ $x = 2 \rightarrow \frac{3x^2 + \epsilon}{x + m} = -1 \Rightarrow m = -3$ $y = f \circ f^{-1}(x) + f^{-1}(x) \Rightarrow x + \frac{3x + \epsilon}{x - 3} = x$
 $\frac{3x + \epsilon}{x - 3} = 0 \Rightarrow 3x + \epsilon = 0 \Rightarrow x = -\frac{\epsilon}{3}$ جواب
 معادله را به دست آوریم
 با فرض $x = 3$

① $f(x - r_n) = x - g(\frac{x}{r})$
 $f(x) = \begin{cases} a \\ -r \end{cases}$ $f^{-1}(x) = \begin{cases} r \\ a \end{cases} \Rightarrow x - g(\frac{x}{r}) = -r \Rightarrow r = g(\frac{x}{r})$ $g(x) = \begin{cases} \frac{x}{r} \\ g^{-1}(x) \end{cases}$
 $f^{-1}(f(x - r_n)) = f^{-1}(-r) \Rightarrow f^{-1}(-r) = x - r_n$
 $\Rightarrow f^{-1}(x) + f^{-1}(-r) = x - r_n + r = \boxed{3}$ جواب

⑨ $f(x) = \frac{x + 4}{x - 1} \Rightarrow f^{-1}(x) = \frac{x + 4}{x - 2} \Rightarrow \frac{x + 4}{x - 1} = \frac{x + 4}{x - 2}$

$\frac{x + 4}{x - 1} = x \Rightarrow x^2 - 2x = x + 4 \Rightarrow x^2 - 3x - 4 = 0$
 $x^2 - 3x - 4 = 0 \Rightarrow x = 4, -1$ $\alpha + \beta = -\frac{b}{a} = 3$ جواب $\boxed{3}$

⑩ $y = x + \epsilon[x]$ $f^{-1}(x) = [y] = [x + \epsilon[x]] \Rightarrow [y] = [x] + \epsilon[x]$
 $\frac{[y]}{\epsilon} = \frac{[x] + \epsilon[x]}{\epsilon} = \frac{[y] - x}{\epsilon}$
 $\frac{[y]}{\epsilon} = \frac{[y] - x}{\epsilon} \Rightarrow x = y - \epsilon[x]$
 $(f - g)(x) = x + \epsilon[x] - x + \frac{\epsilon}{\omega}[x] = \frac{\epsilon}{\omega}[x]$ جواب $\boxed{\frac{\epsilon}{\omega}[x]}$
 $f^{-1}(x) = g(x) = x - \frac{\epsilon}{\omega}[x]$