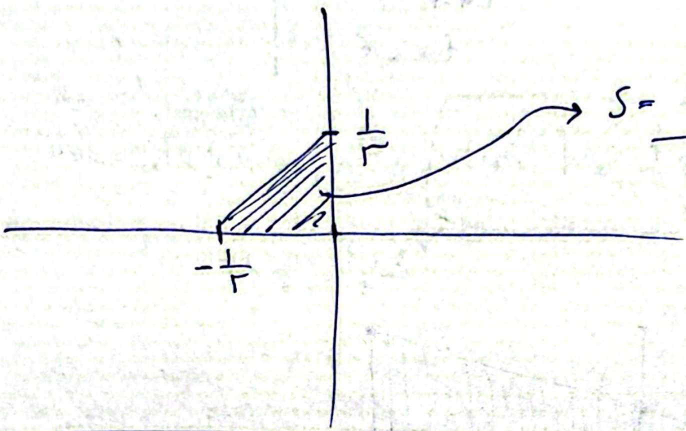
فاطمه رستمیان - دوازدهم

سوال ۱

$2y = 3x - 1 \rightarrow 2x + 1 = 3y \rightarrow y = \frac{2x+1}{3}$

$f^{-1}(x) = \frac{2x+1}{3} \xrightarrow[\text{محورهای مختصات}]{\text{برخورد با}} y = \frac{2(0)+1}{3} = \begin{vmatrix} 0 \\ \frac{1}{3} \end{vmatrix} \begin{vmatrix} -\frac{1}{2} \\ 0 \end{vmatrix}$



$S = \frac{\frac{1}{3} \times \frac{1}{2}}{2} = \frac{\frac{1}{6}}{2} = \frac{1}{12}$

سوال ۲

الف) $y = x^2 - 2x + 2 \quad x \in [1, +\infty)$

$x_s = \frac{-b}{2a} = \frac{2}{2} = 1$ لحاظ کردن x_s ⊗ سمت به تنهایی یک به یک

در حالی که سمت راست و چپ لزوماً یک به یک باشد در سمت تنها یک به یک است ✓

معکوس $y = x^2 - 2x + 1 + 2 \rightarrow y = (x-1)^2 + 2 \rightarrow x - 2 = (y-1)^2$

$\pm\sqrt{x-2} + 1 = y \rightarrow f^{-1}(x) = \pm\sqrt{x-2} + 1$

فقط به خاطر دامنه

$\boxed{f^{-1}(x) = \sqrt{x-2} + 1}$

$f(x)$ برد $= f^{-1}(x)$ دامنه

$\boxed{D_{f^{-1}(x)} = [2, +\infty)}$

$\cancel{1-2}+2 = [2, +\infty)$

ب) $y = x^2 - 2x + 2 \rightarrow -\frac{b}{2a} = \frac{2}{2} = 1 \quad x \in [2, +\infty)$

چون از راس به بعد یک به یک است ✓

$y = x^2 - 2x + 1 + 2 \rightarrow y = (x-1)^2 + 2 \rightarrow x - 2 = (y-1)^2$

معکوس $(+)\sqrt{x-2} + 1 = y \rightarrow \boxed{f^{-1}(x) = \sqrt{x-2} + 1}$

$R_{f(x)} = D_{f^{-1}(x)} \rightarrow 4 - \cancel{4} + 2 = 2 \quad \boxed{[2, +\infty)}$

سوال ۳

$$f(x) = 2x - \sqrt{16 - 4x(4-x)}$$

$$f(x) = 2x - \sqrt{16 - 16x + 4x^2}$$

$$f(x) = 2x - \sqrt{(2x-4)^2}$$

$$f(x) = 2x - |2x-4|$$

$$2\times 3 - |6 - \cancel{4}| = 4$$

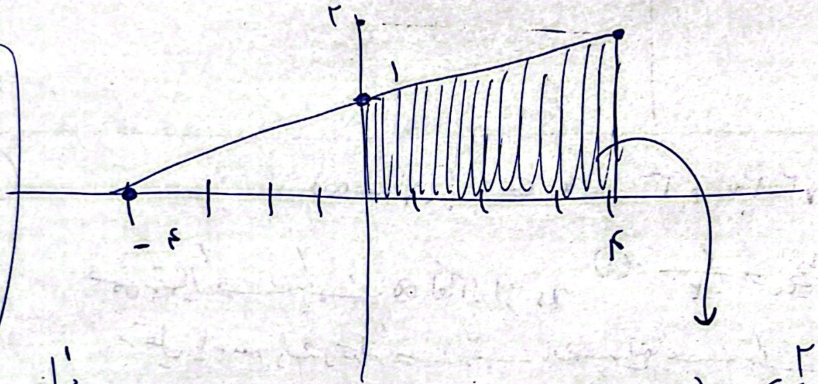
2	
$2x + 2x - 4$	$2x - 2x + 4$
$4x - 4$	4

[illegible] ④

[illegible]

$$y = 4x - 4 \longrightarrow \boxed{\frac{x+4}{4} = f^{-1}(x)}$$

$$\frac{4+4}{4} = \frac{8}{4} = 2$$



$$\frac{(1+2)\times \cancel{4}^{2}}{\cancel{2}} = 3\times 2 = 6$$

سوال ۴

$$y = \frac{x}{\sqrt{x^2-5}} \xrightarrow{\text{توان } 2} y^2 = \frac{x^2}{x^2-5}$$

$$y^2x^2 - 5y^2 = x^2 \longrightarrow y^2x^2 - x^2 = 5y^2 \longrightarrow x^2(y^2-1) = 5y^2$$

$$x^2 = \frac{5y^2}{y^2-1} \longrightarrow y^2 = \frac{5x^2}{x^2-1} \longrightarrow f^{-1}(x)^2 = \frac{5x^2}{x^2-1}$$

$$f^{-1}(x) = \pm\sqrt{\frac{5x^2}{x^2-1}} \longrightarrow f^{-1}(x) = \frac{\sqrt{5}\,x}{\sqrt{x^2-1}}$$

سوال ۵

$$g(x) = \begin{cases} x^2 - 4x + k & x \geq 2 \xrightarrow{2} 4 - 8 + k \quad -4 + k \\ -x^2 + 6x + 2k & x < 2 \rightarrow -4 + 12 + 2k \rightarrow 8 + 2k \end{cases}$$

$$-4 + k \geq 8 + 2k \rightarrow 2k \leq -12 \rightarrow \boxed{k \leq -6}$$

سوال ۷

$$f(x) = \frac{2x + 4}{x + m} \qquad [2 ; -10]$$

$$-10 = \frac{4 + 4}{2 + m} \rightarrow \cancel{-10}^{-1} = \frac{\cancel{10}}{2 + m} \rightarrow -2 - m = 1 \rightarrow -m = 3$$

$$\boxed{m = -3}$$

$$y = \underset{\downarrow \atop x}{f \circ f^{-1}} + f^{-1}(x)$$

$$\cancel{x} + \frac{2x + 4}{x - 2} = \cancel{x} \rightarrow 2x + 4 = 0 \rightarrow 2x = -4$$

$$\boxed{x = -\frac{4}{2}}$$

سوال ۸

$$4g^{-1}(4) + f^{-1}(-2) \qquad f(\underbrace{(2 - 2x)}_{a}) = 2 - g(\underbrace{\tfrac{x}{2}}_{b})$$

$$f(a) = -2 \rightarrow f^{-1}(-2) = a$$

$$g(b) = 4 \rightarrow g^{-1}(4) = b$$

$$4b + a \rightarrow 4(\tfrac{x}{2}) - 2x + 3 = 4(\tfrac{x}{2}) - 2x + 2$$

$$2x - 2x + 2 = 3$$

سوال ۹

$$ab = ? \qquad g(x) = ax + b|x|$$

$$f(x) = 3x + |x|$$

$(a-b)x$	$(a+b)x$

$2x$	$4x$

$$f^{-1} = \frac{x}{2} \qquad f^{-1} = (\tfrac{x}{4})$$

$$a + b = \frac{1}{4}$$

$$a + \cancel{b} = \frac{1}{2} \rightarrow 2a = \frac{3}{4} \rightarrow a = \frac{3}{8} \quad b = -\frac{1}{8} \qquad ab = -\frac{3}{64}$$

Scanned with CamScanner

سوال ۹

$$y=\frac{5x-13}{1-x}\xrightarrow[(0:0)]{\text{قرینه نسبت}}-y=\frac{-5x-13}{+1+x}$$

دو واحد راست

$$y=\frac{5x+13}{x+1}\rightarrow y=\frac{5(x-2)+13}{(x-2)+1}\rightarrow y=\frac{5x+3}{x-1}$$

۳ پایین

$$y=\frac{5x+3}{x-1}-3\rightarrow y=\frac{5x+3-3x+3}{x-1}$$

$$\boxed{y=\frac{2x+6}{x-1}}$$

$$f(x)=\frac{ax+b}{cx+d}\rightarrow f^{-1}(x)=\frac{-dx+b}{cx-a}$$

$$f^{-1}(x)=\frac{x+6}{x-2}$$

$$\frac{x+6}{x-2}=\frac{2x+6}{x-1}\rightarrow 2x^2-4x+6x-12=x^2+5x-6$$

$$x^2-3x-6=0$$

$$x^2-Sx+P=0$$

$$\boxed{S=3}$$

سوال ۱۰

$$f(x)=x+4[x]\rightarrow x=y+4[y]$$

$$y=x-[4y]\rightarrow x=y-4[y]$$

$$\xrightarrow{\text{براکت می‌گیریم}}[x]=[y+4[y]]$$

$$[x]=[y]+4[y]=5[y]$$

$$[y]=\frac{[x]}{5}\rightarrow 4[y]=\frac{4[x]}{5}$$

$$y=x-\frac{4[x]}{5}$$

$$(f-g)(x)\rightarrow f(x)-g(x)\rightarrow \cancel{x}+4[x]-\cancel{x}+\frac{4[x]}{5}$$

$$4[x]+\frac{4[x]}{5}\rightarrow\frac{20[x]+4[x]}{5}\rightarrow\boxed{\frac{24[x]}{5}}$$

$$D_{f^{-1}}=R_f=\{x/x\ldots$$