غزل قاسمی

(1)

$2y = 3x - 1 \xrightarrow{\text{وارون}} 2x + 1 = 3y \to y = \frac{2x+1}{3}$

$\begin{array}{c|c} 0 & \frac{1}{3} \end{array}$

$\begin{array}{c|c} -\frac{1}{2} & 0 \end{array} \to \dfrac{\frac{1}{2} \times \frac{1}{3}}{2} = \frac{1}{12}$

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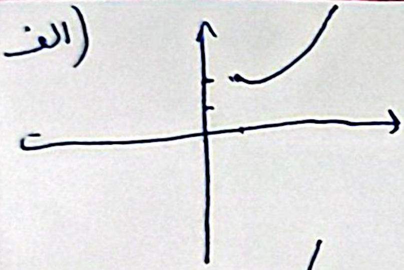
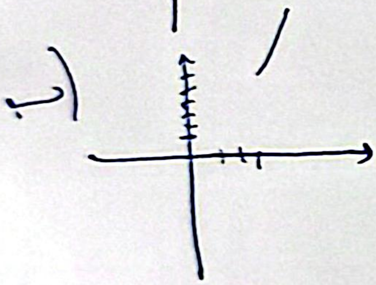
(2)

الف)



یک به یک است

ب)



یک به یک

$\to (x-1)^2 + 2 = y \xrightarrow{\text{وارون}} (y-1)^2 + 2 = x \to y = \sqrt{x-2} + 1$

$R_f \in [3, +\infty)$

$D_f \in [4, +\infty)$

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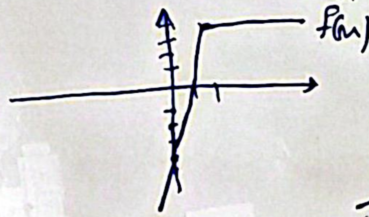
(3)

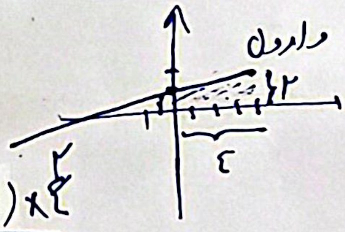
$f(x) = 2x - \sqrt{\underbrace{4x^2 - 16x + 16}_{(2x-4)(2x-4)}} = 2x - |2x-4| \begin{cases} \xrightarrow{x \geq 2} 2x - 2x + 4 = 4 \\ \xrightarrow{x < 2} 2x + 2x - 4 = 4x - 4 \end{cases}$



$y = 4x - 4$: در $x \leq 2$ وارون پذیر است ←

$\xrightarrow{\text{وارون}} x = 4y - 4 \to \frac{x+4}{4} = y$

$R \leq 2$



$6 = \frac{(2+1) \times 4}{2}$

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(4)

$y = \frac{x}{\sqrt{x^2 - 5}} \xrightarrow{\text{وارون}} x = \frac{y}{\sqrt{y^2 - 5}} \xrightarrow{\text{توان ۲}} x^2 = \frac{y^2}{y^2 - 5} \to y^2x^2 - 5x^2 = y^2$ ← یک به یک است

$y = \pm\sqrt{\frac{5x^2}{x^2-1}} \leftarrow y^2 = \frac{5x^2}{x^2-1} \leftarrow y^2(x^2-1) = 5x^2$

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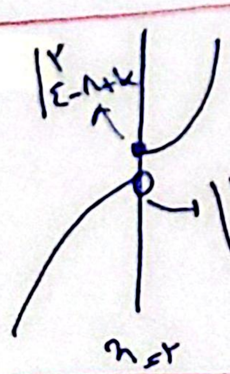
(5)



$-4 + k \geq 8 + 3k$

$-12 \geq 2k$

$-6 \geq k$

$\begin{cases} x^2 - 4x + k : \frac{-b}{2a} \leq 2 \to \frac{4}{2} \leq 2 \checkmark \\ -x^2 + 6x + 3k : \frac{-b}{2a} \geq 2 \quad \frac{-6}{-2} = 3 \checkmark \end{cases}$

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(6)

$x \geq 0 \to 4x = y \xrightarrow{\text{وارون}} \frac{x}{4} = y \; (y \geq 0) \to f(x) \; \frac{3x}{8} \pm \frac{x}{8} \to a = \frac{3}{8}, \; b = \frac{1}{8}$

$x < 0 \to 2x = y \xrightarrow{\text{وارون}} \frac{x}{2} = y \; (y < 0)$

$\frac{3}{8} \times -\frac{1}{8} = \frac{-3}{64}$

$\left|\begin{matrix}2\\-10\end{matrix}\right. \to \frac{10}{2+m}=-10 \to m=-3 \to f(x)=\frac{3x+4}{x-3}$

$fof^{-1}(x)=x$

$f^{-1}(x) \Rightarrow f(x)=y=\frac{3x+4}{x-3} \to xy-3y=3x+4 \to xy-3x=3y+4 \to x(y-3)=3y+4$

$\to x=\frac{3y+4}{y-3} \xrightarrow{\text{وارون}} y=\frac{3x+4}{x-3} \Rightarrow f^{-1}(x)=\frac{3x+4}{x-3}$

$fof^{-1}(x)+f^{-1}(x)=x+\frac{3x+4}{x-3}=\frac{x^2-3x+3x+4}{x-3}=\frac{x^2+4}{x-3}$

$\to \frac{x^2+4}{x-3}=x \to \cancel{x^2}+4=\cancel{x^2}-3x \to -4=3x \to x=\frac{4}{-3}$

$f^{-1}(-2)=t \to f(t)=-2 \to f(\underbrace{3-2x}_{t})=\underbrace{2-g(\tfrac{x}{2})}_{-2} \to g(\tfrac{x}{2})=4 \to g^{-1}(4)=\frac{x}{2}$ (8)

$g^{-1}(4)=m \to g(m)=4 \to f(3-2x)=2-g(\underbrace{\tfrac{x}{2}}_{m}) \to \frac{-f(3-2x)+2}{4}=g(\underbrace{\tfrac{x}{2}}_{m})$

$f(3-2x)=-2 \to f^{-1}(-2)=3-2x \to 3=4g^{-1}(4)+f^{-1}(2)=4\times\frac{x}{2}+3-2x$

(9)

$\xrightarrow[\text{۳ به پایین}]{\text{۲ به راست}} \frac{5(x-2)-13}{1-(x-2)}-3=y \to \frac{5x-10-13}{1-x+2}-3=y \to \frac{5x-23+3x-9}{-x+3}=y$

$\to f(x)=\frac{8x-32}{-x+3}=y$

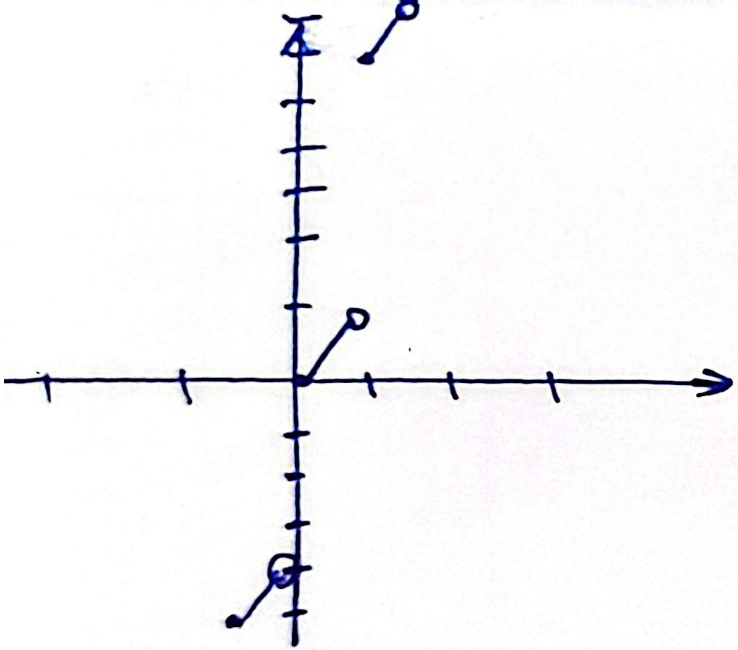
$f^{-1}(x) \Rightarrow y=\frac{8x-32}{3-x} \to 3y-xy=8x-32 \to 3y+32=8x+xy \to \frac{3y+32}{8+y}=x$

$\xrightarrow{\text{وارون}} f^{-1}(x)=y=\frac{3x+32}{8+x}$

$\frac{3x+32}{x+8}=\frac{8x-32}{-x+3} \to 3+\frac{8}{x+8}=-8+\frac{-8}{-x+3} \to 11=\frac{-8}{x+8}-\frac{8}{-x+3}$

$\to 11=\frac{8x-24-8x-64}{-x^2-5x+15} \to \cancel{11}=\frac{-\cancel{88}^{8}}{-x^2-5x+15} \to -x^2-5x+15=-8$

$\to 0=x^2+5x-23 \to \alpha+\beta=-5=-\frac{b}{a}$



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$[-۲,-۱) \to x-۸=y \xrightarrow{\cancel{\text{وارون}}} y=x+۸ \to f-g=-۱۴$

$[-۱,۰) \to x-۴=y \xrightarrow{\text{وارون}} y=x+۴ \to f-g=-۸$

$[۰,۱) \to x=y \xrightarrow{\text{وارون}} y=x \to f-g=۰$

$[۱,۲) \to x+۴=y \xrightarrow{\text{وارون}} y=x-۴ \to f-g=۸$

$[۲,۳) \to x+۸=y \xrightarrow{\text{وارون}} y=x-۸ \to f-g=۱۴$

$f-g(x)=۰$