غزل قاسمی

۱۷/۵

(۱)

$2y=3x-1 \xrightarrow{\text{وارون}} 2x+1=3y \to y=\frac{2x+1}{3}$; $\Big|_{0}^{\frac{1}{2}}$; $\Big|_{0}^{-\frac{1}{2}} \to \frac{\frac{1}{2}\times\frac{1}{3}}{2}=\frac{1}{12}$

۲

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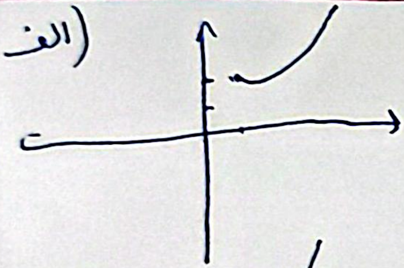
(۲)

الف)

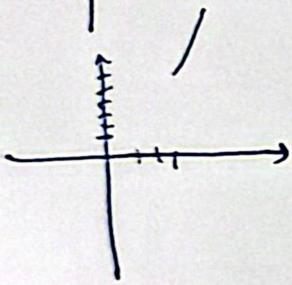


یک به یک نیست

کامل تر راه حل بنویس!

۲

ب)



یک به یک

$\to (x-1)^2+2=y \xrightarrow{\text{وارون}} (y-1)^2+2=x \to y=\sqrt{x-2}+1$

$R_f \in [2,+\infty)$

$D_f \in [1,+\infty)$

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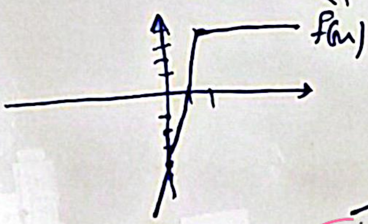
(۳)

$f(x)=2x-\sqrt{4x^2-16x+16}=2x-|2x-4|$; $\xrightarrow{x\geq 2} 2x-2x+4=4$; $\xrightarrow{x<2} 2x+2x-4=4x-4$

$(2x-4)(2x-4)$

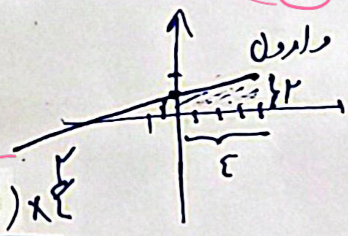
۲



$y=4x-4$: در $x\leq 2$ وارون پذیر است

(وارون) $x=4y-4 \to \frac{x+4}{4}=y$

$R\leq 2$



$6=\frac{(2+1)\times 4}{2}$

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(۴)

$y=\frac{x}{\sqrt{x^2-5}} \xrightarrow{\text{وارون}} x=\frac{y}{\sqrt{y^2-5}} \xrightarrow{\text{توان ۲}} x^2=\frac{y^2}{y^2-5} \to y^2x^2-5x^2=y^2$ یک به یک است

$y=\pm\sqrt{\frac{5x^2}{x^2-1}} \leftarrow y^2=\frac{5x^2}{x^2-1} \leftarrow y^2(x^2-1)=5x^2$

۲

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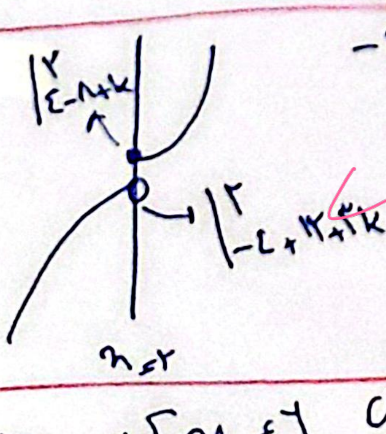
(۵)



$-4+k\geq 8+3k$

$-12\geq 2k$

$-6\geq k$

$\begin{cases} x^2-4x+k: \frac{-b}{2a}\leq 2 \to \frac{4}{2}\leq 2 ✓ \\ -x^2+6x+3k: \frac{-b}{2a}\geq 2 \quad \frac{-6}{-2}=3 ✓ \end{cases}$

۲

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(۶)

$x\geq 0 \to 4x=y \xrightarrow{\text{وارون}} \frac{x}{4}=y \ (y\geq 0) \to f(x)\ \frac{3x}{8}\pm\frac{x}{8} \to a=\frac{3}{8},\ b=-\frac{1}{8}$

$x<0 \to 2x=y \xrightarrow{\text{وارون}} \frac{x}{2}=y$

$(y<0)$

$\frac{3}{8}\times-\frac{1}{8}=\frac{-3}{64}$

۲

$$\left|\begin{matrix}2\\-10\end{matrix}\right. \to \frac{10}{2+m} = -10 \to m=-3 \to f(x) = \frac{3x+4}{x-3}$$

$f\circ f^{-1}(x) = x$ (4)

$$f^{-1}(x) \Rightarrow f(x) = y = \frac{3x+4}{x-3} \to xy - 3y = 3x+4 \to xy - 3x = 3y+4$$

$$x(y-3) = 3y+4$$

$$f^{-1}(x) = \frac{3x+4}{x-3} \Leftarrow y = \frac{3x+4}{x-3} \xleftarrow{\text{جابجایی}} x = \frac{3y+4}{y-3}$$

$$f\circ f^{-1}(x) + f^{-1}(x) = x + \frac{3x+4}{x-3} = \frac{x^2-3x+3x+4}{x-3} = \frac{x^2+4}{x-3}$$

$$x = \frac{-4}{3} \leftarrow -4 = 3x \leftarrow \cancel{x^2}+4 = \cancel{x^2} - 3x \leftarrow \frac{x^2+4}{x-3} = x$$

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(8)

$$f^{-1}(-2) = t \to f(t) = -2 \to f(\underbrace{3-2x}_{t}) = \underbrace{2-g(\tfrac{x}{2})}_{-2} \to g(\tfrac{x}{2}) = 4$$

$$g^{-1}(4) = \frac{x}{2}$$

$$g^{-1}(4) = m \to g(m) = 4 \to f(3-2x) = 2 - g(\underbrace{\tfrac{x}{2}}_{m}) \to \frac{-f(3-2x)+2}{4} = g(\tfrac{x}{2})$$ (3)

$$4\times\frac{x}{2} + 3 - 2x = 3 = 4g^{-1}(4) + f^{-1}(2) \leftarrow f^{-1}(-2) = 3-2x \leftarrow f(3-2x) = -4$$

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(9)

$$\frac{5(x-2)+13}{1-(x-2)} - 3 = y \to \frac{5x-10-13}{1-x+2} - 3 = y \to \frac{5x-23+3x-9}{-x+3} = y$$ (1)

واحد به راست، واحد به پایین

$$-y = \frac{-5x-13}{x+1} \to y = \frac{5x+13}{x+1} \to \frac{5(x-2)+13}{x-2+1} - 3 \quad f(x) = \frac{8x-32}{-x+3} = y$$

$$f^{-1}(x) \Rightarrow y = \frac{8x-32}{3-x} \to 3y - xy = 8x - 32 \to 3y + 32 = 8x + xy \to \frac{3y+32}{8+y} = x$$

$$\frac{5x+3}{x-1} - 3 \to \frac{5x+3-3x+3}{x-1} \to \frac{2x+6}{x-1}$$

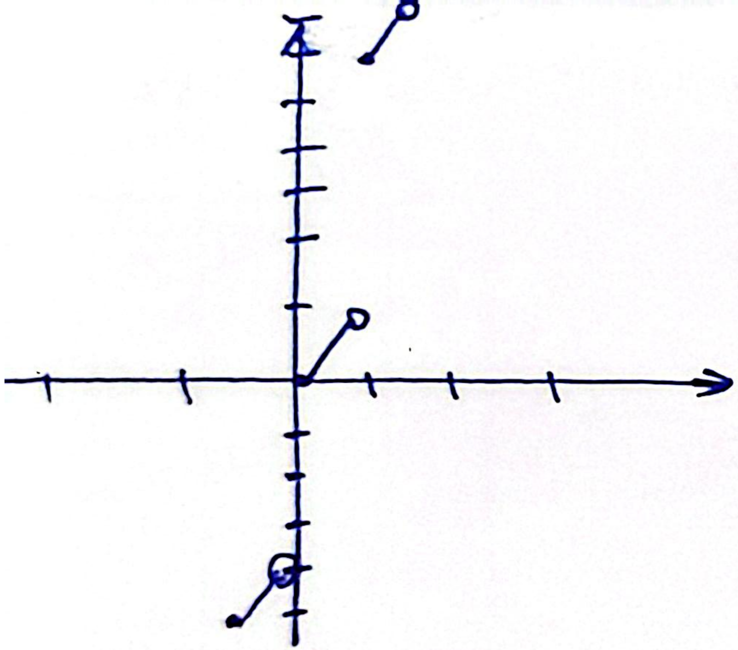
$$f^{-1}(x) = y = \frac{3x+32}{8+x}$$ وارون

$$\frac{3x+32}{x+8} = \frac{8x-32}{-x+3} \to 3 + \frac{8}{x+8} = -8 + \frac{-8}{-x+3} \to 11 = \frac{-8}{x+8} - \frac{8}{-x+3}$$

$$f^{-1}(x) = \frac{x+6}{x-2} \to f(x) = f^{-1}(x) \to x^2 - 3x - 6 = 0,\ -\frac{b}{a} = 3$$

$$-x^2 - 5x + 15 = 8 \leftarrow 11 = \frac{-88}{-x^2-5x+15} \Leftarrow 11 = \frac{8x - 24 - 8x - 64}{-x^2-5x+15}$$

$$0 = x^2 + 5x - 23 \to \alpha + \beta = -5 = -\frac{b}{a}$$



$[-۲,-۱) \to x-۸=y$ ~~وارون~~ $\to y=x+۸ \to f-g=-۱۴$

$[-۱,۰) \to x-۴=y \xrightarrow{\text{وارون}} y=x+۴ \to f-g=-۸$ ۱۰

$[۰,۱) \to x=y \xrightarrow{\text{وارون}} y=x \to f-g=۰$

$[۱,۲) \to x+۴=y \xrightarrow{\text{وارون}} y=x-۴ \to f-g=۸$ ۱۵

$[۲,۳) \to x+۸=y \xrightarrow{\text{وارون}} y=x-۸ \to f-g=۱۶$

$f-g(x) = ۵$

$[y] = ۴[x] + [x] \to \frac{[y]}{۵} = [x]$

$f^{-1}(x) = g(x) \longrightarrow x - \frac{۴}{۵}[x]$

$f-g = x + ۴[x] - x + ۰/۸[x] \to ۴/۸[x]$