

الف) $\xrightarrow{\text{تغییر متغیر}} \left. \begin{matrix} (x, y_1) \in f \\ (x, y_2) \in f \end{matrix} \right\} \rightarrow y_1 = y_2 \Rightarrow \begin{cases} y_1^2 = x^2 + x + 1 \\ y_2^2 = x^2 + x + 1 \end{cases} \rightarrow y_1^2 = y_2^2 \rightarrow y_1 = y_2$ تابع است

ب) $\xrightarrow{\text{مشتق}} x = \frac{\pi}{2} \rightarrow \frac{\pi}{2} \cos y = \sin(\frac{\pi}{2}) \rightarrow \cos y = 0 \rightarrow y = \frac{\pi}{2}, -\frac{\pi}{2}, \frac{3\pi}{2}, \dots$
برای $x = \frac{\pi}{2}$ دارای مقادیر متناهی از y است پس تابع نیست.

ج) $x + y = 2\sqrt{xy} \rightarrow (\sqrt{x} - \sqrt{y})^2 = 0 \rightarrow \sqrt{x} = \sqrt{y} \rightarrow x = y$ برقرار است پس تابع است.

د) $\xrightarrow{\text{مشتق}} x = 0 \rightarrow y^2 + 2y = 0 \rightarrow y(y+2) = 0 \rightarrow y = 0 \rightarrow (0, 0) \in f$
 $y = -2 \rightarrow (0, -2) \in f$
با تعریف y در مشتق است چون $x=0$ برابر دو مقادیر y شد پس تابع نیست.

2) $f(x) = \begin{cases} x^2 - b & ; x < -1 \cup x \geq 2 \\ a + 2x & ; -3 < x < -1 \end{cases} \xrightarrow{x=-1} 1 - b = a - 2 \Rightarrow a + b = 3$

3) $D_f \Rightarrow a - bx - x^2 \geq 0 \rightarrow D_f = [b, a] \rightarrow \begin{matrix} a, b \text{ در } \\ \text{مقادیر متناهی} \end{matrix}$
 $\xrightarrow{\text{مشتق}} \begin{matrix} x=b \\ x=a \end{matrix} \rightarrow \begin{matrix} a - b^2 - b^2 \\ a - ab - a^2 \end{matrix} \rightarrow a(1 - b - a) = 0 \xrightarrow{a \neq 0} 1 - b - a = 0 \rightarrow a + b = 1$

4) $D_f \Rightarrow ax^2 + bx + c > 0 \xrightarrow{D_f = (-\infty, 2)} -(x-2)^2 > 0 \rightarrow -x^2 + 4x - 4 > 0$
 $a = -1 \rightarrow a = -\frac{1}{4}b, b = 4, c = -4 = -b \rightarrow 2(-\frac{1}{4}b) - b - 2b = -\frac{19}{2}b$

5) الف) $|x| - [x] \neq 0 \rightarrow [x] \neq |x| \rightarrow D_f = \mathbb{R} - \mathbb{N}$

ب) $\sqrt{y} = 9 - \sqrt{x} \rightarrow x \geq 0$ ①

جواب مثبت یا دارد پس $9 - \sqrt{x} \geq 0 \rightarrow 9 \geq \sqrt{x} \rightarrow 81 \geq x \rightarrow x \leq 81$ ② $\xrightarrow{\text{مشتق}} D_f = [0, 81]$

الف) $x_{n+1} > 0 \rightarrow x > \frac{1}{x} \rightarrow x \in (\frac{1}{x}, +\infty) - \{1\}$ ①
 $x > 0, x \neq 1$
ب) $\xrightarrow{x} \lg x^{x-1} > 0 \rightarrow x-1 > 1 \rightarrow x > \frac{2}{x}$ ②
ج) $\xrightarrow{x} \lg x^{x-1} > 0 \rightarrow x-1 < 1 \rightarrow x < \frac{2}{x}$ ③
 $\xrightarrow{\text{مشتق}} D_f = (\frac{1}{x}, \frac{2}{x}) \cup (1, +\infty)$

$$\Rightarrow \frac{1}{4+\sqrt{|x|}-|x|} > 0 \rightarrow 4+\sqrt{|x|}-|x| > 0 \xrightarrow{\sqrt{|x|}=t} t+t^2 > 0 \rightarrow \frac{-t}{-1+t} > 0$$

$$\rightarrow -t < t < 2 \rightarrow -\sqrt{|x|} < \sqrt{|x|} < 2 \rightarrow \sqrt{|x|} < 2 \rightarrow |x| < 4 \rightarrow -2 < x < 2 \quad (1)$$

$$\sqrt{|x|} \Rightarrow |x| > 0 \quad (2) \rightarrow (1) \cap (2) \rightarrow D_f = (-2, 2)$$

$$(1) \frac{r_n+r}{n+b} + a > 0 \rightarrow D_{f(1)} = \text{مستقيم}$$

$$(2) n+b \neq 0 \rightarrow n \neq -b \rightarrow D_{f(2)} = \mathbb{R} - \{-b\}$$

چون بازي و مشتق و ... است (1)، (2) و ... است (2)، (3) و ... است (3)

$$f(n) = \begin{cases} r_{n-1} & n > 0 \\ r_{n+1} & n < 0 \end{cases}$$

$$y = \begin{cases} n > 0 & y = \sqrt{r_{n-1}-n+1} = \sqrt{n} \rightarrow n > 0 \quad (1) \\ n < 0 & y = \sqrt{r_{n+1}-n+1} = \sqrt{r_{n+1}} \rightarrow r_{n+1} > 0 \rightarrow n > -\frac{r}{r} \quad (2) \end{cases}$$

$$(1) \cup (2) \rightarrow D_f = \left[-\frac{r}{r}, +\infty\right)$$

$$y = \frac{r \varepsilon n^r + r_{n+1}^r + 1/n + r}{(\varepsilon n^r + \varepsilon_{n+1})^r} = \frac{r(r_{n+1})(r_{n+1})^r}{(r_{n+1})^r} = \frac{r}{r_{n+1}} \rightarrow \text{مستقيم}$$

$$D_f \Rightarrow \varepsilon n^r + \varepsilon_{n+1} \neq 0 \rightarrow (r_{n+1})^r \neq 0 \rightarrow n \neq -\frac{1}{r} \rightarrow D_f = \mathbb{R} - \left\{-\frac{1}{r}\right\}$$

$$rx - a > 0 \rightarrow x > \frac{a}{r} \quad (1)$$

$$1 - \lg^{(r_{n-a})} > 0 \rightarrow 1 > \lg^{r_{n-a}} \rightarrow r_{n-a} < 1 \rightarrow n < \frac{1+a}{r} \quad (2)$$

$$(1) \cap (2) \rightarrow \frac{a}{r} < x < \frac{1+a}{r} \rightarrow D_f = \left(\frac{a}{r}, \frac{1+a}{r}\right) \rightarrow c-b = \frac{1+a}{r} - \frac{a}{r} = \frac{1}{r}$$