

۱۹/۷۵

الف) $y^3 - 2x^2 = 2x - 1$
 تعریف $\begin{cases} y_1^3 - 2x_1^2 = 2x_1 - 1 \\ y_2^3 - 2x_2^2 = 2x_2 - 1 \end{cases} \Rightarrow y_1^3 = y_2^3 \Rightarrow y_1 = y_2$
 تابع است ✓

ب) $x \cos y = y \cos x$
 تابع نیت $x = \frac{\pi}{2}$ مثال
 فرض $\frac{\pi}{2} \cos y = 0 \Rightarrow y = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$

۲) $\frac{x+y}{2} = \sqrt{xy} \Rightarrow x+y = 2\sqrt{xy} \Rightarrow x+y-2\sqrt{xy} = 0$
 $\Rightarrow (\sqrt{x} - \sqrt{y})^2 = 0 \Rightarrow \sqrt{x} = \sqrt{y} \Rightarrow x = y$
 تابع است و همانی ✓

تابع است $x^2y^2 + 2x - y^3 - 2y = 0$
 $x(y^2+2) - y(y^2+2) = 0$
 $(y^2+2)(x-y) = 0 \Rightarrow y^2+2=0$ غیرممکن
 $x-y=0 \Rightarrow x=y$ ✓

$f(x) = \begin{cases} 2x^2 - b & ; |x-1| \geq 2 \\ a + 2x & ; -3 \leq x \leq -1 \end{cases} = \begin{cases} 2x^2 - b & ; x \geq 3 \\ 2x^2 - b & ; x \leq -1 \\ a + 2x & ; -3 \leq x \leq -1 \end{cases}$
 $2(-1)^2 - b = a + 2(-1)$
 $2 - b = a - 2$
 $a + b = 4$ ✓

$f(x) = \sqrt{a - bx - x^2}$ } $D_f = [b, a]$
 $f(x) = \sqrt{-(x-a)(x-b)}$

$a + b = 2 - 1 = 1$

$a + b = -2 - 1 = -3$

$\Rightarrow -x^2 - bx + a = -x^2 + (a+b)x - ab$
 $a+b = -b \Rightarrow a = -2b$
 $a = -ab \xrightarrow{a \neq 0} b = -1$
 $\Rightarrow a = 2 \quad a = 2$

$y = \frac{2}{\sqrt{ax^2 + bx + c}}$ $D_f = (-\infty, 2)$ با توجه به شکل تابع و تعریف تابع درجه دوم
 است بنا بر این $a = 0$

$bx + c > 0 \Rightarrow x > -\frac{c}{b} = 2 \Rightarrow -c = 2b \Rightarrow c = -2b$

$\frac{2a+c-3|b|}{0-2b} = -2b+3b = b$ ✓

با توجه به اینکه b است

الف) $f(x) = \sqrt{\frac{1}{|x| - [x]}}$

$|x| \neq [x]$

$x \geq 0 \quad x \in [x] \quad x \in \mathbb{Z} \quad D_f = \mathbb{R} - \mathbb{Z}^+$
 $x < 0 \quad -x \in [x] \Rightarrow x \in [x] \quad D_f = \mathbb{R} - \mathbb{W}$
 ← اعداد گویا

ب) $f(x) = \sqrt{x} + \sqrt{y} = 9 \quad \sqrt{y} = 9 - \sqrt{x}$
 $\sqrt{x} \Rightarrow x \geq 0 \quad 9 - \sqrt{x} \geq 0 \Rightarrow 9 \geq \sqrt{x} \Rightarrow 81 \geq x$
 $D_f = [0, 81]$ ✓

$$f(x) = \sqrt{\log_{x^2} (x-1)} \quad x^2 - 1 > 0 \rightarrow x > 1$$

$$x > 0 \quad x \neq 1$$

$$\log_{x^2} (x-1) \geq \frac{x-1}{x} \Rightarrow x^2 - 1 \geq 1 \rightarrow x^2 \geq 2 \rightarrow x \geq \sqrt{2} \quad \text{I} \cap \text{II} = [1, +\infty)$$

$$\log_{x^2} (x-1) \geq \frac{x-1}{x} \Rightarrow x^2 - 1 \leq 1 \rightarrow x^2 \leq 2 \rightarrow x \leq \sqrt{2} \Rightarrow x \in (0, \frac{\sqrt{2}}{2}]$$

$$D_f = \left(\frac{1}{\sqrt{2}}, \frac{\sqrt{2}}{2} \right] \cup [1, +\infty)$$

$$b) f(x) = \log \frac{1}{x + \sqrt{x^2 - 1}}$$

$$\frac{1}{x + \sqrt{x^2 - 1}} > 0 \rightarrow x + \sqrt{x^2 - 1} < 0$$

$$-(\sqrt{x^2 - 1} - x)(\sqrt{x^2 - 1} + x) < 0$$

$$x_1, x_2 = \pm 1$$

$$- - 1 + 1 = 0$$

$$x \in (-1, 1)$$

$$y = \sqrt{\frac{x^2 + 2}{x + b}} + a$$

$$D_f = (2, +\infty)$$

با توجه به فرمت (a) ضرب در صورت و مخرج

$$\frac{x^2 + 2}{x + b} + a \geq 0 \Rightarrow \frac{x^2 + 2 + a(x + b)}{x + b} \geq 0 \Rightarrow \frac{x(x + a) + 2 + ba}{x + b} \geq 0$$

$$a = -2$$

$$b = -2$$

$$\frac{1}{x - 2} \geq 0$$

$$f(x) = \begin{cases} x^2 - 1; [x] > \varepsilon \\ \varepsilon x + 2; [x] \leq \varepsilon \end{cases} = \begin{cases} x^2 - 1; x > a \\ \varepsilon x + 2; x < a \end{cases}$$

$$\text{I } x > a \Rightarrow y = \sqrt{x^2 - 1 - x + 1} = \sqrt{x} \Rightarrow x \geq 0 \quad \text{II} \Rightarrow \text{I} \cap \text{II} = [a, +\infty)$$

$$\text{III } x < a \Rightarrow y = \sqrt{\varepsilon x + 2 - x + 1} = \sqrt{\varepsilon x + \varepsilon} \Rightarrow x \geq -\frac{\varepsilon}{\varepsilon} \quad \text{IV} \Rightarrow \text{III} \cap \text{IV} = \left[-\frac{\varepsilon}{\varepsilon}, a\right)$$

$$D_f = \left[-\frac{\varepsilon}{\varepsilon}, +\infty\right)$$

$$y = \frac{x^2 \varepsilon + 2x + 1}{(\varepsilon x^2 + \varepsilon x + 1)^2} = \frac{x(1x^2 + 2x + 1)}{(x^2 + 1)^2} = \frac{x(x+1)^2}{(x^2 + 1)^2}$$

$$(x+1)^2 \neq 0 \Rightarrow x \neq -1$$

$$= \frac{1}{\varepsilon(x+1)^2}$$

$$D_f = \mathbb{R} - \left\{-\frac{1}{\varepsilon}\right\}$$

$$y = \sqrt{1 - \log(x - a)}$$

$$x - a > 0 \rightarrow x > \frac{a}{p}$$

$$1 - \log(x - a) \geq 0$$

$$D_f = (b, c] = \left[\frac{a}{p}, \frac{1+a}{p}\right] \rightarrow -\log(x - a) \geq -1 \rightarrow \log(x - a) \leq 1$$

$$\rightarrow x - a \leq 10$$

$$x \leq \frac{1+a}{p}$$

$$c - b = \frac{1+a}{p} - \frac{a}{p} = \frac{1}{p}$$