حل تمرین A صفحه ۱۲

Year:........ Month:........ Day:........ Subject:........

(1)

الف) $y^3 - 2x^2 = 2x - 1 \to y^3 = 2x^2 + 2x - 1$

$(x_1, y_1) \in f$, $(x_2, y_2) \in f$ $\xrightarrow{x_1 = x_2} y_1^3 = y_2^3 \to y_1 = y_2$ تابع است

ب) $x \cos y = y \cos x$ تابع نیست

$x = \frac{\pi}{2} \to \frac{\pi}{2} \cos y = y \cancel{\cos(\frac{\pi}{2})} \to \cos y = 0 \to y = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$

ج) $\frac{x+y}{2} = \sqrt{xy}$ تابع است

$x + y = 2\sqrt{xy} \to x - 2\sqrt{xy} + y = 0 \to (\sqrt{x} - \sqrt{y})^2 = 0 \to x = y$

د) $xy^2 + 2x - y^3 - 2y = 0$ تابع است $\to xy^2 + 2x = y^3 + 2y$

$x(y^2 + 2) = y^3 + 2y \to x = \frac{y^3 + 2y}{y^2 + 2}$

$(x_1, y_1) \in f$, $(x_2, y_2) \in f$ $\xrightarrow{x_1 = x_2} y_1 = y_2 \to \begin{cases} x = \frac{y_1^3 + 2y_1}{y_1^2 + 2} \to y_1(y_1^2 + 1) \\ x = \frac{y_2^3 + 2y_2}{y_2^2 + 2} \to y_2(y_2^2 + 1) \end{cases}$ $y_1 = y_2$

(2)

$f(x) = \begin{cases} 2x^2 - b \; ; \; |x-1| \geq 2 \\ a + 2x \; ; \; -3 \leq x \leq -1 \end{cases}$

$x - 1 \geq 2 \to x \geq 3$

$-2 \geq x - 1 \to -1 \geq x$

این دو در $x = -1$ یکسان اند

$2(-1)^2 - b = a - 2 \to 2 - b = a - 2$

$a + b = 4$

ب) $f(x) = \sqrt{x} + \sqrt{y} = 9 \to \sqrt{y} = 9 - \sqrt{x} \to 9 - \sqrt{x} \geq 0 \to 9 \geq \sqrt{x}$

$D_f = [0, 81] \leftarrow 81 \geq x$

(3)

الف) $f(x) = \frac{1}{\sqrt{|x| - [x]}} \to |x| \neq [x] \quad x \in \mathbb{R} - \{\mathbb{Z}^+, 0\}$

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(۳) تابع باید max داشته باشد

$(-x+b)(x-a) = -x^2 + ax + bx - ab$

$-x^2 + ax + bx - ab = -x^2 - bx + a$

$-x^2 + (a+b)x - ab = -x^2 - bx + a \to a+b = -b \to a - 1 = 1 \to a = 2$

$\downarrow a = -ab \to b = -1$

$a + b = 2 - 1 = 1$

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(٤) $y = \dfrac{2}{\sqrt{ax^2+bx+c}}$ $\quad D_f = (-\infty, 2)$ دامنه برای تابع خطی صدق میکند

$a = 0$

$2b + c = 0 \to c = -2b$

$3a + c - 3|b| \to 3(0) - 2b - 3(-b)$

$a = 0$

$-2b + 3b = b$

$b \to$ منفی است

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(۶) الف) $f(x) = \sqrt{\log_x \frac{3x-1}{x}} \longleftrightarrow x \geq 3x - 1 \geq 1 \to x \geq \frac{2}{3}$

$\hookrightarrow 0 < x < 1 \to 3x - 1 \leq 1 \to 3x \leq 2 \to x \leq \frac{2}{3}$

$3x - 1 > 0 \to x > \frac{1}{3}$

$x > 0$

$x \neq 1$

اشتراک همه $\to$ $\left(\frac{1}{3}, \frac{2}{3}\right] \cup (1, +\infty)$

ب) $f(x) = \log \dfrac{1}{4 + \sqrt{|x|} - |x|} \to \dfrac{1}{4 + \sqrt{|x|} - |x|} > 0 \to 4 + \sqrt{|x|} - |x| > 0$

فرض $\sqrt{|x|} = t$ $\quad -t^2 + t + 4 > 0 \to$ $\quad -2 \quad 3$ $\quad - (+) -$

$-9 < x < 9 \leftarrow \sqrt{|x|} < 3$ $\quad -2 < \sqrt{|x|}$ همواره برقرار

ARYA

$$\sqrt{\frac{3x+2+ax+ab}{x+b}} = \sqrt{\frac{(3+a)x+ab+2}{x+b}} \qquad \frac{-ab-2}{3+a} \quad -b$$

$+ \;|\; - \;|\; +$

(7)

$b=-2$, $a=-1 \leftarrow 2 = \frac{-ab-2}{3+a} = -b$ یا

$I \cup II : x \geq \frac{-4}{3}$

$2x-1-x+1 \geq 0 \to x \geq 0 \xrightarrow{\cap} x \geq 5 \quad I$

$$f(x) = \begin{cases} 2x-1 \; ; \; [x] > 4 \Rightarrow x \geq 5 \\ 4x+3 \; ; \; [x] \leq 4 \Rightarrow x < 5 \end{cases}$$

$f(x)-x+1 \geq 0$

(8)

$4x+3-x+1 \geq 0 \to 3x+4 \geq 0 \to x \geq \frac{-4}{3} \xrightarrow{\cap} 5 > x \geq -\frac{4}{3} \quad II$

$$y = \frac{24x^3+36x^2+18x+3}{(4x^2+4x+1)^2} = 9x+3$$

(9)

$4x^2+4x+1 \neq 0 \to x \neq -\frac{1}{2} \to x \in \mathbb{R} - \{-\frac{1}{2}\}$

$2x-a > 0 \to x > \frac{a}{2}$

(10)

$1 \geq \log_{10}^{2x-a} \longrightarrow 10 \geq 2x-a \longrightarrow \frac{10+a}{2} \geq x > \frac{a}{2}$

$\frac{10+a}{2} \geq x$

$(\frac{a}{2}, \frac{10+a}{2}]$

$(b, c]$

$c-b = \frac{10}{2}+\frac{a}{2}-\frac{a}{2} = 5$