

الف) $y^3 - 2x^2 - 2x - 1 \Rightarrow y^3 = 2x^2 + 2x + 1 \Rightarrow y = \sqrt[3]{2x^2 + 2x + 1}$ تابع است

ب) $x \cos y = y \cos x \xrightarrow{\text{تفاضل}} x = \frac{\pi}{2} \Rightarrow \frac{\pi}{2} \cos y = 0 \Rightarrow y = k\pi + \frac{\pi}{2}$
 $k \in \mathbb{Z} \Leftarrow$ تابع نیست

ج) $\frac{x+y}{2} = \sqrt{xy}$; $x, y > 0$ $\xrightarrow{\text{تفاضل}} x^2 + y^2 + 2xy = 4xy \Rightarrow x^2 + y^2 - 2xy = 0 \Rightarrow (x-y)^2 = 0 \Rightarrow x-y=0 \Rightarrow x=y$ تابع است

د) $xy^2 + 2x - y^3 - 2y = 0 \Rightarrow x(y^2 + 2) = y(y^2 + 2) \Rightarrow x = y$ تابع است

$f(x) = \begin{cases} 2x^2 - b & ; |x-1| \geq 2 \\ a + 2x & ; -3 \leq x \leq -1 \end{cases}$ $|x-1| \geq 2 \xrightarrow{\text{تفاضل}} \begin{matrix} x-1 \geq 2 \rightarrow x \geq 3 \\ x-1 \leq -2 \rightarrow x \leq -1 \end{matrix} \Rightarrow \begin{matrix} x \leq -1 \\ x \geq 3 \end{matrix}$

$\Rightarrow x = -1 \Rightarrow 2x^2 - b = a + 2x \Rightarrow 2 - b = a - 2 \Rightarrow \boxed{a+b=4}$

$f(x) = \sqrt{a-bx-x^2}$; $D_f = [b, a]$

$\Rightarrow f(a) = 0 \Rightarrow \sqrt{a-ab-a^2} = 0 \Rightarrow a-ab-a^2 = 0 \xrightarrow{a \neq 0} 1-b-a=0$

$\Rightarrow 1 = a+b$

$y = \frac{2}{\sqrt{ax^2+bx+c}}$, $D_f = (-\infty, 2)$ $3a+c-3|b|=?$

الف. مضاعف داشته

$\sqrt{ax^2+bx+c} = \sqrt{-(x-2)^2} \Rightarrow -x^2 + 4x - 4 = ax^2 + bx + c$

$\Rightarrow a = -1$, $b = 4$, $c = -4 \Rightarrow 3a+c-3|b| = -3-4-12 = -19$

حساب b
 $b = 4 \Rightarrow \boxed{\text{جواب} = -\frac{19}{4}b}$

$$\text{الف) } f(x) = \sqrt[3]{\frac{1}{|x| - [x]}} \rightarrow |x| - [x] \neq 0 \Rightarrow |x| \neq [x]$$

$$\Rightarrow D_f = \mathbb{Z}^+ \cup \{0\}$$

$$\text{ب) } f(x) = \sqrt{x} + \sqrt{y} = 9 \rightarrow \sqrt{y} = 9 - \sqrt{x} \Rightarrow y = 81 - 18\sqrt{x} + x$$

$$\Rightarrow y = x - 18\sqrt{x} + 81 \xrightarrow{\sqrt{x}=t} t^2 - 18t + 81 = 0 \Rightarrow (t-9)^2 = 0 \Rightarrow t=9$$

$$\Rightarrow x = 81 \rightarrow \frac{1}{-1+} \xrightarrow{y \geq 0} D_f = [81, +\infty)$$

$$\text{الف) } f(x) = \sqrt[n]{\log_n^{n-1} x} \rightarrow \left. \begin{array}{l} n-1 > 0 \\ x > 0 \end{array} \right\} \Rightarrow x > \frac{1}{n}, x > 0 \quad \text{I}$$

$$\log_n^{n-1} x \geq 0 \rightarrow n-1 \geq 1 \Rightarrow x \geq \frac{1}{n} \quad \text{II}$$

$$\text{I} \cap \text{II} \xrightarrow{D_f} x \geq \frac{1}{n}$$

$$\text{ب) } f(x) = \log \frac{1}{4 + \sqrt{|x|} - |x|} \rightarrow \frac{1}{4 + \sqrt{|x|} - |x|} > 0 \rightarrow 4 + \sqrt{|x|} - |x| > 0$$

$$\xrightarrow{\sqrt{|x|}=t} -t^2 + t + 4 > 0 \rightarrow -(t-4)(t+1) > 0 \Rightarrow t < 4 \quad \checkmark$$

$$\xrightarrow{t=4} |x| = 9 \Rightarrow x = \pm 9 \quad \xrightarrow{\begin{array}{l} 4 + \sqrt{|x|} - |x| > 0 \\ -9 < 4 \\ -1 < 4 \end{array}} D_f = (-9, 9)$$

$$y = \sqrt{\frac{x^2 + 1}{x+b} + a} ; D = (r, +\infty) \rightarrow \frac{x^2 + 1 + ax + ab}{x+b} > 0 \rightarrow$$

$$\frac{x(x+1+a) + (1+ab)}{x+b} \geq 0 \xrightarrow{\text{with } x \geq 0} x = \frac{-d}{c} \Rightarrow x = -b \xrightarrow{x=r} r = -b \Rightarrow \boxed{b = -r}$$

$$y = \sqrt{f(x) - x + 1} ; f(x) = \begin{cases} x-1 ; [x] > r \\ x+1 ; [x] \leq r \end{cases} \rightarrow \begin{cases} x-1 ; x \geq \Delta \\ x+1 ; x < \Delta \end{cases}$$

$$x \geq \Delta \rightarrow \sqrt{x-1-x+1} = \sqrt{x} \Rightarrow x \geq 0$$

$$\left\{ \begin{array}{l} x < \Delta \rightarrow \sqrt{x+1-x+1} = \sqrt{x+1} \Rightarrow x+1 \geq 0 \Rightarrow x \geq -1 \\ \end{array} \right. \rightarrow \boxed{y \text{ defined } = x \geq 0}$$

$$y = \frac{rx^r + ryx^r + 1x + r}{(ex^r + ex + 1)^r} = \frac{r(rx+1)^r}{(rx+1)^r} = r(rx+1); x \neq -\frac{1}{r} \quad 9$$

$$\Rightarrow y \text{ axis} = \mathbb{R} - \left\{-\frac{1}{r}\right\}$$

$$y = \sqrt{1 - \log(rx-a)}; D = (b, c] \quad 10$$

$$\rightarrow 1 - \log(rx-a) \geq 0 \Rightarrow \log(rx-a) \leq 1 \Rightarrow rx-a \leq 10 \Rightarrow x \leq \frac{10+a}{r}$$

$$; rx-a > 0 \Rightarrow x > \frac{a}{r} \Rightarrow D = \left(\underbrace{\frac{a}{r}}_b, \underbrace{\frac{10+a}{r}}_c \right]$$

$$\bullet c - b = \frac{10+a}{r} - \frac{a}{r} = \omega$$