

$$f(-n) - n f(n) = 2|n| + 1 \xrightarrow{x=n}$$

$$f(n) + n f(-n) = 2|n| + 1 \xrightarrow{\text{درج اولیہ، قطر 3}}$$

$$\begin{cases} -n f(-n) + n^2 f(n) = -2|n| + 1 \rightarrow n \\ n f(-n) + f(n) = 2|n| + 1 \end{cases}$$

$$(n^2 + 1) f(n) = 2|n| + 1 - 2|n| + 1 - n$$

$$f(n) = \frac{2|n| + 1 - 2|n| + 1 - n}{n^2 + 1} \begin{cases} n \geq 0 & \frac{-2n^2 + n + 1}{n^2 + 1} \\ n < 0 & \frac{2n^2 - 3n + 1}{n^2 + 1} \end{cases}$$

$$f(-n) + f(n) = n^2 - 3n + 4 \xrightarrow{n=0} \therefore f(0) = 4 \rightarrow f(0) = 3$$

$$f(n) = n^2 - 3n + 4$$

$$f(n^2) = (n^2)^2 - 3(n^2) + 4 = n^4 - 3n^2 + 4$$

$$y = \frac{n^2 + n + 1}{n - 1}$$

$$y_n - y = n^2 + n + 1$$

$$n^2 + (1 - y)n + y = 0$$

$$\Delta = (1 - y)^2 - 4(y + 1) = y^2 - 4y - 3$$

$$\Delta \geq 0 \rightarrow y^2 - 4y - 3 \geq 0 \rightarrow (y - 7)(y + 1) \geq 0 \rightarrow \frac{-1}{+} \frac{7}{-}$$

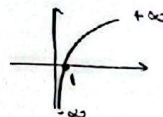
$$R = (-\infty, -1] \cup [7, +\infty)$$

$$\text{الف) } y = 2x^2 - 2x + 1 \xrightarrow{a > 0} \begin{cases} x_{\min} = \frac{-b}{2a} = \frac{1}{2} \\ y_{\min} = \frac{-14}{4} \end{cases} \rightarrow R_f = \left[\frac{-14}{4}, +\infty \right)$$

$$\text{ب) } y = \sqrt{-x^2 + 4x - 4} \xrightarrow{a < 0} \begin{cases} x_{\max} = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_{\max} = 2 \end{cases} \rightarrow R_f = \sqrt{[-\infty, 4]} = [0, 2]$$

$$\text{ج) } \sqrt{n^2 - 2n + 1} \rightarrow R_f = \sqrt{R} = [0, +\infty)$$

$$\text{د) } y = \log(n^2 - 4n + 4) \rightarrow R_f = (-\infty, +\infty)$$



$$y = \frac{n^2 + 1}{n - 4} \rightarrow R = \left\{ \frac{4}{3} \right\} \rightarrow R_f = \mathbb{R} - \left\{ \frac{4}{3} \right\}$$

$$y = \sqrt{\frac{n^2 + 1}{n - 4}} \rightarrow R_f = \sqrt{\mathbb{R} - \left\{ \frac{4}{3} \right\}} = [0, +\infty) - \left\{ \frac{4}{3} \right\}$$

$$y = \frac{n^2 + 4}{n^2 + 3} = \frac{n^2 + 3}{n^2 + 3} + \frac{1}{n^2 + 3} = 1 + \frac{1}{n^2 + 3} \xrightarrow{\frac{1}{n^2 + 3} > 0} R = \left[\frac{4}{3}, +\infty \right)$$

درون کثرت مقدار $\sqrt{n^2 + 3}$ کی مقدار
داشتہ ہو گا۔ یہ ممکن ہے کہ سب کثرت
مقدار برابر نہ ہوں: $\sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

$$y = n^2 + \frac{1}{n^2 + 4} = n^2 + 4 + \frac{1}{n^2 + 4} - 4 \xrightarrow{\frac{1}{n^2 + 4} > 0} R = \left[\frac{1}{4}, +\infty \right)$$

کثرت مقدار $n^2 + 4$ کی مقدار
داشتہ ہو گا۔ یہ ممکن ہے کہ سب کثرت
مقدار برابر نہ ہوں: $4 + \frac{1}{4} - 4 = \frac{1}{4}$

$$y = \frac{3x^2 + 4}{x^2 - 1}$$

برداشت اولی: $y = x^2 - y = 3x^2 + 4 \rightarrow x^2(y - 3) = y + 4 \rightarrow x = \sqrt{\frac{y+4}{y-3}}$

$$\frac{y+4}{y-3} \geq 0 \rightarrow \frac{-4}{3} \leq y < 3 \rightarrow R = (-\infty, -\frac{4}{3}] \cup (3, +\infty)$$

برداشت دوم: $y = \frac{3x^2 + 4}{x^2 - 1}$ $\begin{matrix} x \rightarrow +\infty \\ x \rightarrow 0 \end{matrix}$ $\begin{matrix} 3 \\ -4 \end{matrix}$ $\xrightarrow[\text{مقرنس}]{\text{مخرج مساوی}} R = (-\infty, -\frac{4}{3}] \cup (3, +\infty)$

چون $y = 3$ هرگز حاصل نمی شود
و $y = -\frac{4}{3}$ این مقدار رو به دست آوریم

$$f(x) = \frac{4 \sin x + 3}{\sin x - 3}$$

$$\begin{matrix} x=1 \\ x=-1 \end{matrix} \rightarrow \begin{matrix} -3 \\ -\frac{1}{3} \end{matrix}$$

$$\xrightarrow[\text{مخرج مساوی}]{\text{مخرج مساوی}} R = [-3, -\frac{1}{3}]$$

$$y = \left| \frac{3x-1}{x} \right|$$

چون قدر مطلق داریم
 $y \geq 0$ (II)

$$\frac{3x-1}{x} = y \rightarrow 3x-1 = xy \rightarrow x(3-y) = 1$$

$$x = \frac{1}{3-y} \rightarrow 3-y \neq 0 \rightarrow y \neq 3 \quad (I)$$

$$\text{I} \cup \text{II} \cup \text{III} \rightarrow R = [0, +\infty)$$

$$\frac{3x-1}{x} = -y \rightarrow 3x-1 = -xy \rightarrow x(3+y) = 1$$

$$x = \frac{1}{3+y} \rightarrow \text{همیشه جواب داریم} \quad (II)$$

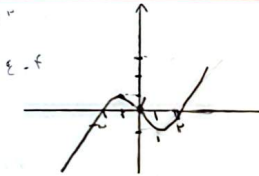
$$\sqrt{\frac{9x-1}{-x+3}} = \sqrt{\frac{9(x-1)}{3(x-1)}} = \sqrt{\frac{9}{3}} = \frac{3}{1} \rightarrow R = \left\{ \frac{3}{1} \right\}$$

$x \neq 1$

$$R = \left\{ \frac{3}{1} \right\}$$

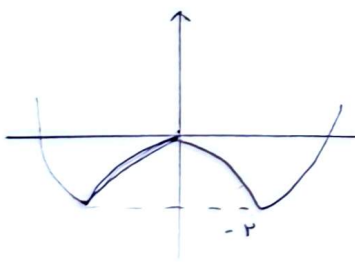
$$y = x|x| - 2x \rightarrow y = x^2 - 2x \quad x > 0$$

$$y = -x^2 - 2x \quad x < 0$$

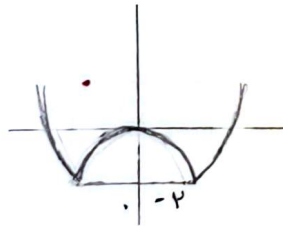


$$R = \mathbb{R}$$

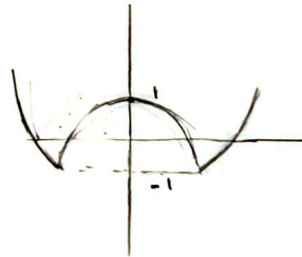
$$y = |f(|x|) + 1|$$



$$f(|x|)$$



$$f(|x|) + 1$$



$$|f(|x|) + 1|$$

