

$$f(x) = \begin{cases} x_{n-1} & ; [n] > f \\ f_{n+1} & ; [n] \leq f \end{cases} \Rightarrow f(x) = \begin{cases} x_{n-1} & ; x \geq \Delta \\ f_{n+1} & ; x < \Delta \end{cases} \quad (2)$$

$$f(x) - x + 1 \geq 0 \quad \begin{cases} 1) \text{ if } x \geq \Delta \Rightarrow x \geq 0 \Rightarrow [\Delta, +\infty) \\ 2) \text{ if } x < \Delta \Rightarrow x_{n+1} \geq 0 \Rightarrow x \geq -\frac{f}{p} \Rightarrow [-\frac{f}{p}, \Delta) \end{cases} \Rightarrow D = [-\frac{f}{p}, +\infty)$$

$$y = \frac{2fx^3 + 34x^2 + 18x + 2}{(x^2 + x + 1)^2} = \frac{3(18x^3 + 12x^2 + 4x + 1)}{(x^2 + x + 1)^2} = \frac{3(2x+1)^3}{(x^2 + x + 1)^2} \quad (2)$$

$$= \frac{3}{x^2 + 1} \Rightarrow D = \mathbb{R} - \left\{ -\frac{1}{2} \right\}$$

$$f(x) = \begin{cases} \frac{x^3 + 2x^2 - x - 2}{x^2 - 1} & ; x \neq \pm 1 \\ \frac{ax + b}{x + 1} & ; x = \pm 1 \end{cases} \Rightarrow \frac{x^3(x+2) - (x+2)}{x^2 - 1} = \frac{(x+2)(x^2 - 1)}{x^2 - 1} = x + 2 \quad (2)$$

$$\text{if } x \neq \pm 1 \Rightarrow g(x) = f(x) \Rightarrow x + c = x + 2 \Rightarrow c = 2 \quad (1) \Rightarrow g(x) = x + 2$$

$$\text{if } x = 1 \Rightarrow f(x) = g(x) \Rightarrow \frac{a+2}{1+b} = 3 \Rightarrow a+2 = 3+3b \Rightarrow a = 3b+1$$

$$\text{if } x = -1 \Rightarrow f(x) = g(x) \Rightarrow \frac{-a+2}{-1+b} = 1 \Rightarrow 2-a = b-1 \Rightarrow a = 3-b$$

$$\Rightarrow 3b+1 = 3-b \Rightarrow b = \frac{1}{2} \quad (2) \quad \Rightarrow b+c = 2 + \frac{1}{2} = \frac{5}{2}$$

$$f(x) = g(x) \Rightarrow \frac{(x-1)(x+1)}{x^2 + ax + b} = \frac{x+1}{x-c} \quad \text{با ضرب کردن } f(x) \text{ و } g(x) \text{ در } x=1 \text{ و } x=-1 \text{ به دست می آید}$$

$$1-c=0 \Rightarrow c=1 \quad (2)$$

$$x^2 + ax + b = (x-1)(x-1) = (x-1)^2 = x^2 - 2x + 1 \Rightarrow a = -2, b = 1$$

$$\Rightarrow a+b+c = -2+1+1 = 0 \quad \checkmark$$



$$g(x) = f(x+1) + f(x-1) = (x-1)^2(x) + (-x-1)^2(-x) = x((x-1)^2 - (x+1)^2) \\ = x(x-1-x-1)(x-1+x+1) = x(-2)(2x) = -4x$$

$$f(1-x) = \begin{cases} \sqrt{x} + 1 & ; x \leq 0 \\ \sqrt{x} & ; x > 0 \end{cases} \quad \textcircled{1} 1-x = -1 \Rightarrow x = 2 \Rightarrow f(-1) = 2 \\ \textcircled{2} 1-x = 1 \Rightarrow x = 0 \Rightarrow f(1) = 1 \\ \Rightarrow f(-1) + f(1) = 2 + 1 = 3$$

$$x + \sqrt{x} = 4 \quad \sqrt{x} = t \quad t^2 + t - 4 = 0 \Rightarrow (t-2)(t+3) = 0 \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4 \\ \Rightarrow f(4) = (f)^2 + 1(f) = 16 + 4 = 20$$

$$f(x-1) + f(1-x) = (x-1)^2 + (x-1) + (1-x)^2 + (1-x) = (x-1)^2 + (x-1) - (x-1)^2 - (x-1) = 0$$

$$D_f \Rightarrow x - \frac{1}{x} \geq 0 \Rightarrow x \geq \frac{1}{x} \\ D_g \Rightarrow b - \frac{1}{x} \geq 0 \Rightarrow x \leq \frac{1}{b} \Rightarrow D_{f \cap g} = \left[\frac{1}{b}, \frac{1}{x} \right] \rightarrow \frac{1}{x} = 1 = \frac{1}{b} \Rightarrow a = \frac{1}{x}, b = \frac{1}{x} \\ (xf - g)(1) = xf(1) - g(1) = x\sqrt{1-x} + xk - 1 - \sqrt{1-x} + k = k \\ \Rightarrow -2k = 1 \Rightarrow k = -\frac{1}{2} \\ \Rightarrow xa - \frac{1}{x} - k = x - \frac{1}{x} + \frac{1}{2} = \frac{2x^2 - 2 + 1}{2x} = \frac{2x^2 - 1}{2x}$$

$$D_f \Rightarrow m - x \geq 0 \Rightarrow m \geq x \Rightarrow D_f = (-\infty, m] \\ D_g \Rightarrow x + 1 \geq 0 \Rightarrow x \geq -1 \Rightarrow D_g = [-1, \infty) \\ \Rightarrow D_f \cap D_g = [-1, m] = [-1, 1] \\ f(x) - g(x) = 4\sqrt{x} \Rightarrow x + 1 - 4\sqrt{x} = 4\sqrt{x} \Rightarrow x = 8\sqrt{x} - 1 \\ \Rightarrow m + n = 1 + 8\sqrt{x} - 1 = 8\sqrt{x}$$