

۱) $y = \sqrt{f(n) - n + 1} \rightarrow f(n) - n + 1 \geq 0$

$f(n) = \begin{cases} n-1 & ; [n] > 1 \rightarrow n \geq 0 \\ n-1^2 & ; [n] \leq 1 \rightarrow n < 0 \end{cases}$

$n \geq 0 \rightarrow n-1-n+1 \geq 0 \rightarrow 0 \geq 0 \rightarrow n \geq 0$

$n < 0 \rightarrow n-1^2-n+1 \geq 0 \rightarrow n^2 \geq -1$

$n^2 \geq -1 \rightarrow n \geq -1$

$\rightarrow D_y = [-1, +\infty)$

۲) $y = \frac{1^2 n^2 + 1^2 n^2 + 1(n+1)}{(n^2 + (n+1)^2)} = \frac{2n^2 + n + 1}{n^2 + n^2 + 2n + 1} = \frac{2n^2 + n + 1}{2n^2 + 2n + 1}$

$= \frac{1}{n+1}$

$\rightarrow n+1 \neq 0 \rightarrow n \neq -1$

$D_f = \mathbb{R} \setminus \{-1\}$

۳) $f(n) = \begin{cases} \frac{n^2 + n^2 - a - 1}{n^2 - 1} & ; n \neq \pm 1 \\ \frac{a+1}{a+b} & ; n = \pm 1 \end{cases}$

$g(n) = n + c$

$n = 0 \rightarrow f(0) = \frac{0+0-a-1}{0-1} = \frac{-a-1}{-1} = a+1$

$g(0) = 0 + c = c$

$a+1 = c \rightarrow c = a+1$

$n = 1 \rightarrow f(1) = \frac{a+1}{1+b} = \frac{a+1}{1+b}$

$g(1) = 1 + c = 1 + a + 1 = a + 2$

$\frac{a+1}{1+b} = a + 2$

$a+1 = (a+2)(1+b)$

$a+1 = a+2+ab+2b$

$0 = 1+ab+2b$

$ab+2b = -1$

$b(a+2) = -1$

$b = \frac{-1}{a+2}$

$n = -1 \rightarrow f(-1) = \frac{-a+1}{b-1} = \frac{1-a}{b-1}$

$g(-1) = -1 + c = -1 + a + 1 = a$

$\frac{1-a}{b-1} = a$

$1-a = a(b-1)$

$1-a = ab-a$

$1 = ab$

$b = \frac{1}{a}$

$b = \frac{-1}{a+2}$

$b = \frac{1}{a}$

$\frac{-1}{a+2} = \frac{1}{a}$

$-a = a+2$

$-a = a+2$

$-2a = 2$

$a = -1$

$b = \frac{1}{-1} = -1$

۴) $f(n) = \frac{n+1}{n-c}$

$g(n) = \frac{n^2-1}{n^2+an+b}$

① $\rightarrow (n-c)^2 = n^2 + an + b$

② $\rightarrow \frac{n+1}{n-c} = \frac{n^2-1}{(n-c)(n-1)}$

① $\rightarrow (n-c)(n-1) = (n-c)^2 = n^2 + an + b$

$n-c, n-1 \rightarrow c=1$

$n^2 + an + b = n^2 + 1n + 1$

$a=1, b=1$

$a+b+c = 1+1+1 = 3$

$$\begin{aligned}
 \omega) \quad f(n) &= (n-1)^n (n-1) \quad f(n+1) = n(n+1)^n (n+1-1) + (1-n)^n (1-n-1) \\
 &= (n-1)^n (n) + (1-n)^n (-n) = (n^2 - n^2 + 1)(n) - (n^2 + n^2 - 1)(-n) \\
 &= n^2 - n^2 + 1n - n^2 - n^2 - 1n = -1n^2
 \end{aligned}$$

$$\gamma) \quad f(x) = \begin{cases} \sqrt{x} + 1 & x \leq 0 \\ \sqrt{x} & x > 0 \end{cases} \quad f(-1) + f(1) = 1 + 1 = 2$$

$$f(-1) \Rightarrow 1 - 1 = -1 \rightarrow 1 + 1 = 2 \rightarrow f(-1) = 1 + 1 = 2$$

$$f(1) \Rightarrow 1 - 1 = 0 \rightarrow 1 + 0 = 1 \rightarrow f(1) = 1$$

$$\begin{aligned}
 \nu) \quad f(x + \sqrt{x}) &= x^2 + 1 \quad f(4) \rightarrow 4 + \sqrt{4} = 6 \rightarrow 6^2 = 36 \\
 f(4) &= \frac{4^2}{4} + \frac{1 \cdot 4}{4} = 4 + 1 = 5 \\
 \text{for } t &= 4, \quad t + \sqrt{t} = 6 \rightarrow (t + \sqrt{t})^2 = 36 \rightarrow t + \sqrt{t} = 6 \\
 \sqrt{t} &= 2 \rightarrow t = 4
 \end{aligned}$$

$$\begin{aligned}
 \eta) \quad f(n) &= n^2 + n \\
 f(n-1) + f(1-n) &= (n-1)^2 + (n-1) + (1-n)^2 + (1-n) \\
 &= n^2 - 2n + 1 + n - 1 + 1 - 2n + 1 + n - 1 = 0
 \end{aligned}$$

$$\begin{aligned}
 \theta) \quad f(x) &= \sqrt{x^2 - 1} + k - 1 \quad f(1) = \sqrt{1 - 1} + k - 1 = k - 1 \\
 g(x) &= \sqrt{b - x} + k \quad g(1) = \sqrt{b - 1} + k \\
 f \circ g &= f(g(x)) \rightarrow f(\sqrt{b - x} + k) = \sqrt{(\sqrt{b - x} + k)^2 - 1} + k - 1 \\
 &= \sqrt{b - x + 2k\sqrt{b - x} + k^2 - 1} + k - 1 \\
 &= \sqrt{b - x} + k - 1 + k - 1 = 2k - 2 + \sqrt{b - x} \\
 &= \sqrt{b - x} + k - 1
 \end{aligned}$$

$$\begin{aligned}
 \iota) \quad f(x) &= \sqrt{x - 1} + n \quad g(x) = \sqrt{x + 1} \quad D_{f \circ g} = [-1, 1] \\
 D_{f \circ g} &= D_g \cap D_f = [-1, \infty) \cap [1, \infty) = [1, \infty) \\
 D_{f \circ g} &= [-1, 1] \rightarrow D_f = [-1, 1] \rightarrow x - 1 \geq 0 \rightarrow x \geq 1 \\
 &\quad x \leq 1 \\
 f \circ g(x) &= 4\sqrt{x} \\
 \sqrt{x - 1} + n &= 4\sqrt{x} \rightarrow n = 4\sqrt{x} - \sqrt{x - 1} \\
 n + 1 &= 4\sqrt{x} - \sqrt{x - 1} + 1 = 4\sqrt{x} - \sqrt{x - 1} + 1
 \end{aligned}$$