

مسئله ضابطی / واریتم دقت B - ۳

$$f(x) = \begin{cases} 2x-1 & ; [x] > 1 \rightarrow [x] > 1 \rightarrow x > 2 \\ 2x^2 & ; [x] \leq 1 \rightarrow [x] \leq 1 \rightarrow x \leq 2 \end{cases}$$

1

$$y = \sqrt{f(x) - x + 1} \rightarrow f(x) \geq x - 1 \rightarrow \begin{cases} 2x-1 \geq x-1 \rightarrow x \geq 0 \rightarrow x \geq 2 \\ 2x^2 \geq x-1 \rightarrow 2x^2 - x + 1 \geq 0 \end{cases}$$

$\Delta = (-1)^2 - 4 \cdot 2 \cdot 1 = 1 - 8 = -7 < 0$

$D_f = (-\frac{1}{2}, +\infty)$

$$y = \frac{2x^3 + 2x^2 + x + 1}{(2x^2 + 2x + 1)^2} = \frac{2x^3 + 2x^2 + x + 1}{(2x+1)^2} = \frac{2x(2x+1)}{(2x+1)^2} = \frac{2x}{2x+1}$$

2

$$y = \frac{2x}{2x+1}$$

$D_f = \mathbb{R} - \{-1/2\}$

$$f(x) = \begin{cases} \frac{x^3 + 2x^2 - x - 2}{x^2 - 1} & ; x \neq \pm 1 \\ \frac{a+b}{x+b} & ; x = \pm 1 \end{cases}$$

$f(1) = \frac{a+b}{1+b} = 2 \rightarrow a+b = 2(1+b) \rightarrow a+b = 2+2b \rightarrow a = b+2$

$f(-1) = \frac{a+b}{-1+b} = -1 \rightarrow a+b = -1(-1+b) \rightarrow a+b = 1-b \rightarrow a = -2b+1$

$b+2 = -2b+1 \rightarrow 3b = -1 \rightarrow b = -1/3 \rightarrow a = 5/3$

$g(x) = x + c \rightarrow (x+c)(x^2-1) = x^3 + cx^2 - x - c \rightarrow x^3 + cx^2 - x - c = x^3 + 2x^2 - x - 2 \rightarrow c = 2$

3

$$f(x) = \frac{x^2 + 1}{x-1}$$

$$g(x) = \frac{x^2 - 1}{x^2 + ax + b}$$

$a = -2, b = 1$

4

$$f(x) = (x-2)^2(x-1) \rightarrow f(1-x) = ((1-x)-2)^2((1-x)-1) = (-x-1)^2(-x) = -x(x+1)^2$$

B

$$f(x+1) + f(1-x) = (x+1-2)^2(x+1-1) + (-x-1)^2(-x-1) = x^2 + (-x-1)^2(-x-1) = x^2 - (x+1)^2(x+1) = x^2 - (x+1)^3$$

Elegance

$$x^2 - (x+1)^3 = x^2 - (x^3 + 3x^2 + 3x + 1) = -x^3 - 2x^2 - 3x - 1 = -1(x^3 + 2x^2 + 3x + 1)$$

6

$$f(1-x) = \begin{cases} \sqrt{x+1} & x \leq 0 \\ \sqrt{x} & x > 0 \end{cases}$$

$x \leq 0 \rightarrow f(1-x) = \sqrt{x+1} \Rightarrow x = -1 \rightarrow f(1) = 1$
 $x > 0 \rightarrow f(1-x) = \sqrt{x} \Rightarrow x = 1 \rightarrow f(1) = 1$

$$f(-1) + f(1) = 1 + 1 = 2$$

7

$$f(x + \sqrt{x}) = 2x^2 + 5x \rightarrow 2x^2 + 11x = 24$$

$$f(x) = ?$$

$$x + \sqrt{x} = 4 \rightarrow x + \sqrt{x} - 4 = 0$$

$$(\sqrt{x} + 3)(\sqrt{x} - 1) = 0 \rightarrow \sqrt{x} = 1 \rightarrow x = 1$$

$$f(1) + f(1)$$

8

$$f(x) = \frac{x^2 + x}{x(x+1)}$$

$$f(1-x) = \frac{(1-x)^2 + (1-x)}{(1-x)(1-x+1)}$$

$$f(x-1) = \frac{(x-1)^2 + (x-1)}{(x-1)(x-1+1)}$$

$$f(x) + f(1-x) + f(x-1) = 0$$

9

$$f - g = (1+k) \rightarrow \frac{f}{g} = 1 \rightarrow \frac{f}{g} = 1 \rightarrow f = g$$

$$g(x) = \sqrt{b - \epsilon x} + \frac{1}{k} \rightarrow b - \epsilon x \geq 0 \rightarrow b \geq \epsilon x$$

$$f(x) = \sqrt{\epsilon x - \frac{1}{k}} + k - 1 \rightarrow \epsilon x - \frac{1}{k} \geq 0 \rightarrow \epsilon x \geq \frac{1}{k}$$

$$f - g = \sqrt{\epsilon x - \frac{1}{k}} + k - 1 - \sqrt{b - \epsilon x} + \frac{1}{k} = k - 1 \rightarrow -k - 1 \leq k \rightarrow -k - 1 \leq k$$

$$Df \times g = [-1, 1]$$

$$Df \cap Dg$$

$$f - g(x) = \sqrt{b - \epsilon x} + \frac{1}{k} - \sqrt{\epsilon x - \frac{1}{k}} - k + 1 = \sqrt{b - \epsilon x} - \sqrt{\epsilon x - \frac{1}{k}} - k + \frac{1}{k} + 1$$

$$f(x) = \sqrt{mx} + n \rightarrow mx \geq 0 \rightarrow m \geq 0 \rightarrow m \geq 0 \rightarrow m \geq 0 \rightarrow m \geq 0$$

$$g(x) = \sqrt{2x+1} \rightarrow 2x+1 \geq 0 \rightarrow 2x \geq -1 \rightarrow x \geq -\frac{1}{2}$$

Elegance