(تألیف حورا حقیقی)

۱)

$D_{gof(x)} = \{x \mid x \in D_f \cap f(x) \in D_g\}$

$D_f = \begin{matrix} ① & x-1>0 & x>1 \\ ② & \log_2^{x-1} \geq 0 & x-1 \geq 1 & x \geq 2 \end{matrix} \Big\} ① \cap ② = [2, +\infty)$

$D_g = \sqrt{\underbrace{-(x-2)^2}_{\text{همواره منفی}}} \geq 0 \rightarrow x = 2 \quad \text{یا} \quad \sqrt{\log_2^{x-1}} = 2 \quad \log_2^{x-1} = 4$

$x-1 = 16$

$x = 17$

$D_{gof} = ① \cap ② = \{17\}$

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۲)

~~$fog(x) = gof(x)$~~

$2(x^2 - 3x + 8) - 5 = (3x-5)^2 - 3(3x-5) + 8$

~~$2x^2 - 6x + 16 - 5 = 9x^2 + 25 - 30x - 9x + 15 + 8$~~

$2x^2 - 25x + 33 = 0 \qquad \alpha^2 + \beta^2 = S^2 - 2P = 100 - 2 \times \frac{33}{2} = 43$

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۳)

~~$gof = 5x^2 + 11$~~

~~$f(x) = (\frac{x}{4})t \quad \frac{5}{4}t^2 + 11 \Big\} \Rightarrow g(x) = \frac{5}{4}x^2 + 11$~~

کمترین مقدار $g(x-7) = \frac{5}{4}(x-7)^2 + 11 \Rightarrow y_{min} = 11$

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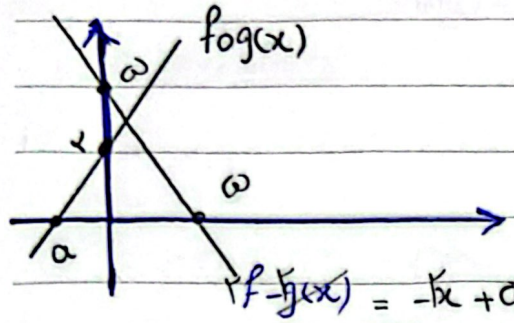
۴)

$f(x) = 3x - 4 \qquad 3(g(x)) - 4 = 3x^2 - 9x - 1$

$fog(x) = 3x^2 - 9x - 1 \qquad 3g(x) = 4x^2 - 6x + 3$

$gof(x) = (3x-5)^2 = 9x^2 - 30x + 25 \qquad g(x) = x^2 - 2x + 1 = (x-1)^2$

(۵



$fog(x) = ۲(۳x-۱)+۴$

$x=۰ \Rightarrow y=۲$ ✓

$y=۰ \Rightarrow x = ۲(۳x-۱) = -۴$

$۳x-۱ = -۲$

$x = \frac{-۱}{۳} = a$

$۲f-۳g(x) = -۱۲x+۵\times۲$

$۲g(x) - ۳f(x) = -۱۴$

$-f(x) = -۲x-۴ \quad f(x)=۲x+۴ \quad g(x)=۳x-۱$

$۲x+۴-g = -x+۵$

$۳x-۱$

(۶

$f(x) = ۹^x$

$g(x) = \log_{۳}(x) > ۰$ ①

$fog(x) = ۹^{\log_{۳}(x)} = x^۲$

$gof(x) = \log_{۳} ۹^x = ۲x$

$x^۲ \leq ۲x \quad ۰ \quad ۲$

$x^۲-۲x \leq ۰ \quad x(x-۲)\leq ۰$

② $\frac{۰ \quad ۲}{+ \quad - \quad +}$

① ∩ ② $۰<x\leq ۲ \Rightarrow \{۱,۲\}$

(۲۰۲۵)

(۷

$f\left(\underbrace{x-\frac{۲}{x}}_{t}\right) = x^۲+x-۴-\frac{۲}{x}+\frac{۴}{x^۲}$ $\quad t^۲ = x^۲+\frac{۴}{x^۲}-۴ \quad +t = x^۲+x-۴-\frac{۲}{x}+\frac{۴}{x^۲}$

$\Rightarrow f(x) = x^۲+x$

(۸

$D_{fog(x)} = \{x \mid \underbrace{x\in D_g}_{\mathbb{R}} \cap \underbrace{g(x)\in D_f}_{x>۰}\}$

$x(۸-x)>۰$ $\quad \frac{۰ \quad ۸}{- \quad + \quad -}$ $\quad ۰<x<۸ = D_{fog} = A$

$\log \frac{۸x-x^۲}{۴}$ $\quad \frac{-b}{۲a} = \frac{۸}{۲} = ۴ \quad ۳۲-۱۶ = ۱۶$

$= ۴$ $\quad (-\infty, ۴] = B$

$A \cap B = (۰,۴] \Rightarrow ۱,۲,۳,۴$ جواب ④

(۹

$fof(x) = 1 - 4f^2(x)$

$g(x) = x - 1$

$h(x) = x^3 + 2x + 1$

$f(\underbrace{f(x)}_{t}) = 1 - 4\underbrace{f^2(x)}_{t^2}$ $\quad f(x) = 1 - 4x^2$

$h(g(x)) = (x-1)^3 + 2(x-1) + 1 = 1 - 4x^2$

$\Rightarrow (x-1)[(x-1)^2 + 2] = -4x^2$

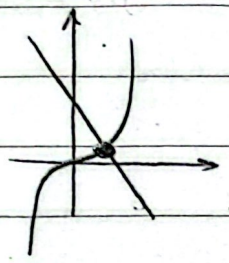
$(x^2 - 2x + 1)$

$-4x^2 = x^3 - 2x^2 + x - x^2 + 2x - 1 + 2x - 2$

$x^3 + 5x - 3^3 = 0$

یک جواب دارد

$x^3 = -5x + 3$



(۱۰

$f(x) = x - \frac{4}{x}$

$f(x) = x - \frac{4}{x} = 3$

$x^2 - 3x - 4 = 0 \quad (x-4)(x+1) = 0$

$gof(x) = ax^2 + 2x$

$g(f(x)) = ax^3 + 2x \rightarrow x = 4 \quad 64a + 8 = 1$

$a = 0$

$\rightarrow x = -1$

$-a - 2 = 1$

$a = -10$