$f(x) = \sqrt{\log_2^{(x-1)}}$ $g(x) = \sqrt{-x^2+4x-4}$ $D_{gof(x)}$

$D_f : \log_2^{(x-1)} \geq 0 \rightarrow (x-1) \geq 1 \rightarrow x \geq 2$ (1) $D_g : -x^2+4x-4 \geq 0$

$x-1 > 0 \rightarrow x > 1$ (2) $-(x-2)^2 \geq 0 \rightarrow (x-2)^2 \leq 0$ (2) $\rightarrow x=2$

$1 \cap 2 \longrightarrow x \geq 2$ $D_{gof} = \{x \mid x \in D_{f(x)} \; ; \; f(x) \in D_{g(x)}\}$ $x=17$

$\underbrace{x \geq 2}_{}$ (1) $\sqrt{\log_2^{(x-1)}} = 2 \rightarrow \log_2^{(x-1)} = 4$ $x-1=16$

$D_{gof} = (1) \cap (2) = \{17\}$

$f(x) = 2x-5$ $g(x) = x^2-3x+8$ $fog(x) = gof(x)$

مجموع مربعات ریشه‌های معادله؟

$f(g(x)) = 2(x^2-3x+8) - 5 = 2x^2-6x+16-5 = 2x^2-6x+11$

$g(f(x)) = (2x-5)^2 - 3(2x-5) + 8 = 4x^2+25-20x-6x+15+8 = 4x^2-26x+48$

$\alpha^2+\beta^2 = S^2 - 2P$

$2x^2-6x+11 = 4x^2-26x+48$ $100 - 2(\frac{37}{2})$

$0 = 2x^2-20x+37 \longrightarrow 0 = x^2 - 10x + \frac{37}{2}$

$0 = x^2 - Sx + P$ $100 - 37 = 63$

$gof(x) = 5x^2+11$ $f(x) = 2x$ $g(x-7) = ?$

$g(2x) = 5x^2+11 \rightarrow g(t) = 5(\frac{t}{2})^2 + 11 = \frac{5t^2}{4} + 11 \rightarrow g(x) = \frac{5x^2}{4} + 11$

$2x = t \rightarrow x = \frac{t}{2}$ $g(x-7) = \frac{5(x-7)^2}{4} + 11$

$\underbrace{\frac{5(x^2-14x+49)}{4}}_{\min 0} + 11 =$

$\boxed{\min g(x-7) = 11}$

$f(x) = 3x-4$ $fog(x) = 3x^2-6x-1$ $gof(x) = ?$

$f(g(x)) \Rightarrow$

$3g(x) - 4 = 3x^2-6x-1 \rightarrow 3g(x) = 3x^2-6x+3 \rightarrow g(x) = x^2-2x+1$

$gof(x) \Rightarrow (3x-4)^2 - 2(3x-4) + 1 = 9x^2-24x+16-6x+8+1 =$

$gof(x) = 9x^2-30x+25$

$2g(x) = 3f(x) - 14$ $f(x) = ax+b$ $g(x) = cx+d$ fog , $f-g$

هم f هم g خطی است

$f(x) - g(x) = ax+b-cx-d = (a-c)x+b-d = -x+5$

$2cx+2d = 3ax+3b-14$ $a-c=-1$ $\begin{cases} b-d=5 \\ 2d = 3b-14 \end{cases}$ $f-g(x) = -x+5$

$2c = 3a$ $2d = 3b-14$ $a - \frac{3}{2}a = -1$

$c = \frac{3}{2}a$ $\boxed{c=3 \; , \; a=2}$ $-\frac{1}{2}a = -1$ $b=4 \;\; d=-1$

$\rightarrow f(x) = 2x+4$
$\rightarrow g(x) = 3x-1$ $\Big\} \rightarrow f(g(x)) = 2(3x-1)+4 = 6x+2 \xrightarrow[\text{با محور x}]{\text{محل برخورد}} 6x+2=0 \rightarrow x = -\frac{1}{3}$

$a = -\frac{1}{3}$

[illegible] - پاسخ تشریحی تکلیف ۸

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$f(x) = 9^x \qquad g(x) = \log_3^x$

$f\circ g(x) \leq g\circ f(x)$

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$f\circ g(x) = f(\log_3^x) = 9^{\log_3^x} = x^{\log_3^9} = x^2$

$g\circ f(x) = \log_3^{9^x} = \log_3^{3^{2x}} = 2x$

$x^2 \leq 2x$

$x^2 - 2x \leq 0$

$x(x-2) \leq 0 \longrightarrow$ ۰ و ۱ و ۲ جواب صحیح

$\begin{array}{c|c|c|c|c} & + & 0 & - & 2 & + \end{array}$

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$f(x)$

$f\left(\frac{x^2-2}{x}\right) = x^2 + x - 4 - \frac{2}{x} + \frac{4}{x^2}$

$f\left(x - \frac{2}{x}\right) = \left(x - \frac{2}{x}\right)^2 + x - \frac{2}{x}$

$f(t) = t^2 + t$

$f(x) = x^2 + x$

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$R_g = (-\infty, 16]$

$g(x) = 8x - x^2 \quad , \quad f(x) = \log_2^x$

$A \to D_{f\circ g}$

$B \to R_{f\circ g}$

$A \cap B = ?$

$f\circ g(x) = f(8x - x^2) = \log_2^{8x-x^2} \qquad \log_2^{16} = 4$

$(0, 8) \cap (-\infty, 4] = (0, 4]$ ۴ عضو صحیح

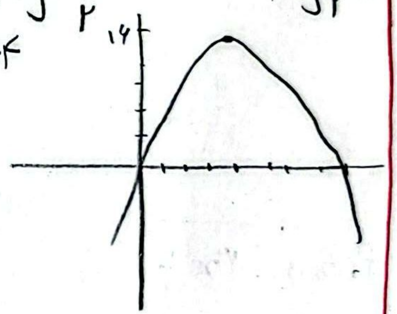
$A = \{x \mid x \in D_g ; g(x) \in D_f\} \longrightarrow (0, 8)$

$\quad\; \mathbb{R} \qquad\qquad 8x - x^2 > 0$

$x(8-x) > 0$

$\begin{array}{c|c|c|c|c} & - & 0 & + & 8 & - \end{array}$

$B =$ برد g ورودی f $\longrightarrow$ برد f برد $f\circ g$ $\longrightarrow (-\infty, 4]$



۹

$h(x) = x^3 + 3x + 1) \qquad g(x) = x - 1 \qquad f\circ f(x) = 1 - 3f^2(x)$

$(x+1)^3$

$f(f(x)) = 1 - 3f^2(x)$

$h(g(x)) = f(x)$

$f(x) = 1 - 3x^2 \leftarrow f(t) = 1 - 3t^2$

$((x-1)^3 + 2(x-1) + 1 = x^3 - 3x^2 + 3x - 1 + 2x - 2 + 1 = x^3 - 3x^2 + 5x - 2$

$x^3 - \cancel{3x^2} + 5x - 2 = 1 - \cancel{3x^2} \rightarrow x^3 + 5x - 3 = 0$

دلتای مشتق منفی است پس عبارت اصلی یک ریشه دارد

$3x^2 + 5 = 0 \longrightarrow \Delta y' < 0$

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$g(3) = 8$

$g\circ f(x) = ax^2 + 2x \qquad f(x) = x - \frac{4}{x}$

$f(x) = 3 \leftarrow g(f(x)) = ax^2 + 2x$

$f(x) = 3 = x - \frac{4}{x}$

$g(3) = 8$

$3x = x^2 - 4$

$ax^2 + 2x = 8$

$0 = x^2 - 3x - 4$

$\hookrightarrow x = 4 \to 16a + 8 = 8 \to a = 0$ ✓

$\hookrightarrow x = -1 - a - 2 = 8 \to a = -10$ ✗

$x = 4 \qquad x = -1$