

$$D_{g \circ f} = \{x \in f(D_f) \mid f(x) \in D_g\}$$

$$f(x) = \sqrt{\log_p^{x-1}} \Rightarrow x-1 > 1 \Rightarrow x > 2 \quad (1) \quad \log_p^{x-1} > 0 \Rightarrow x-1 > 1 \Rightarrow x > 2 \quad (2)$$

$$(1) \cap (2) \rightarrow D_f = (2, +\infty) \quad (A)$$

$$g(x) = \sqrt{-x^2 + 4x - 4} \Rightarrow -x^2 + 4x - 4 \geq 0 \Rightarrow x^2 - 4x + 4 \leq 0 \Rightarrow (x-2)^2 \leq 0$$

لذا جواب منبسط است بهر آنکه $x=2$ در آن جا به یزای سطر ۱ صفری شود

$$\Rightarrow D_g = \{2\} \Rightarrow \sqrt{\log_p^{x-1}} = 2 \Rightarrow \log_p^{x-1} = 4 \Rightarrow x-1 = 2^2 = 4 \Rightarrow x = 1V \quad (B)$$

$$(A) \cap (B) \rightarrow D_{g \circ f} = (2, +\infty) \cap \{1V\} = \{1V\}$$

$$f \circ g(x) = 2(x^2 - 4x + 4) - \omega = 2x^2 - 4x + 4 - \omega = 2x^2 - 4x + 11 \quad (1)$$

$$g \circ f(x) = (2x - \omega)^2 - 3(2x - \omega) + 4 = 4x^2 - 4\omega x + 2\omega^2 - 4x + 1\omega + 4$$

$$\Rightarrow g \circ f(x) = 4x^2 - 4\omega x + 4\omega^2 - 4x + 1\omega + 4 \quad f \circ g(x) = g \circ f(x) \Rightarrow 2x^2 - 4x + 11 = 4x^2 - 4\omega x + 4\omega^2 - 4x + 1\omega + 4$$

$$\Rightarrow 2x^2 - 4\omega x + 11 = 0 \Rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{4\omega \pm \sqrt{16\omega^2 - 44}}{4} \quad (\Delta = b^2 - 4ac)$$

$$\Rightarrow x = \frac{4\omega \pm 2\sqrt{4\omega^2 - 11}}{4} = \frac{4\omega \pm 2\sqrt{4\omega^2 - 11}}{4} \Rightarrow \frac{4\omega \pm 2\sqrt{4\omega^2 - 11}}{4} = \frac{4\omega \pm 2\sqrt{4\omega^2 - 11}}{4}$$

$$= \frac{100 + 44 + 100 + 44 - 40\sqrt{4\omega^2 - 11}}{4} = \frac{200 + 44 - 40\sqrt{4\omega^2 - 11}}{4} = \frac{244 - 40\sqrt{4\omega^2 - 11}}{4}$$

$$g \circ f(x) = \omega x^2 + 11 \quad f(x) = 2x = t \Rightarrow x = \frac{t}{2}$$

$$\Rightarrow g(t) = \omega \left(\frac{t}{2}\right)^2 + 11 = \frac{\omega t^2}{4} + 11 \Rightarrow g(x) = \frac{\omega x^2}{4} + 11 \Rightarrow g(x-v) = \frac{\omega (x-v)^2}{4} + 11$$

$$\Rightarrow g(x-v) = \frac{\omega x^2 - 2\omega xv + \omega v^2}{4} + 11 \Rightarrow$$

با معادله درجه ۲ در صورت قرار بار صفر است

$$\Delta = b^2 - 4ac = 0 \Rightarrow a > 0 \Rightarrow \text{تابع } b \text{ min است} \Rightarrow \text{بسیار است } [0, +\infty) \Rightarrow \text{بسیار است}$$

$$\min(g(x-v)) = \frac{0}{4} + 11 = 11$$

بسیار است

$$\hookrightarrow g(x-v) \text{ بسیار است}$$

$$f \circ g(x) = x^p - 4x - 1 \quad f(x) = x^p - 4$$

(۴)

$$\Rightarrow f(g(x)) = x^p - 4 \Rightarrow x^p - 4 = x^p - 4x - 1 \Rightarrow x^p = x^p - 4x + 3$$

$$\Rightarrow g = x^p - 4x + 1 = (x-1)^p \Rightarrow g(x) = (x-1)^p$$

$$\Rightarrow g \circ f(x) = (x^p - 4 - 1)^p = (x^p - 5)^p = x^{p^2} - 10x^p + 5$$

$$\Rightarrow pf(x) - pg(x)$$

(۵)

$$\left. \begin{aligned} p(f-g)(x) &= -p(x) + q(p) \\ pg(x) - pf(x) &= -1f \end{aligned} \right\} \Rightarrow -f(x) = -px - 4 \Rightarrow f(x) = px + 4$$

$$px + 4 - g = -x + q \Rightarrow g(x) = x^p - 1$$

$$\Rightarrow f \circ g(x) = p(x^p - 1) + 4 \quad x=0 \rightarrow y=p \checkmark$$

$$y=0 \rightarrow p(x^p - 1) = -4 \Rightarrow x = -\frac{1}{p} = a \Rightarrow a = -\frac{1}{p}$$

$$f(x) = a^x = p^{px} \quad g(x) = \log_p x \Rightarrow f \circ g(x) = p^{p \log_p x} = p^{\log_p x^p} = x^p$$

(۶)

$$\Rightarrow g \circ f(x) = \log_p p^{px} = px \log_p p = px \quad \hookrightarrow f \circ g(x) = (x^p)^{\log_p p} = x^p$$

$$\Rightarrow f \circ g(x) \neq g \circ f(x) \Rightarrow x^p \neq px \Rightarrow x^p - px \neq 0 \Rightarrow x(x-p) \neq 0$$

$$\Rightarrow \frac{0}{+} - \frac{p}{-} + \frac{p}{+} \Rightarrow x \in (0, p] \rightarrow \{1, p\}$$

$$f\left(\frac{x^p - p}{x}\right) = x^p + x - p - \frac{p}{x} + \frac{p}{x^p} = x^p - p + \frac{p}{x^p} + x - \frac{p}{x}$$

(۷)

$$\Rightarrow f\left(x - \frac{p}{x}\right) = x^p - p + \frac{p}{x^p} + x - \frac{p}{x} = \left(x - \frac{p}{x}\right)^p + \left(x - \frac{p}{x}\right)$$

$$\Rightarrow f(x) = x^p + x$$

$$\Rightarrow f(x) = x^p + x$$

$$f(n) = \log_p n, g(n) = \Lambda n - n^2 \Rightarrow f(g(n)) = \log_p -n^2 + \Lambda n \quad (A)$$

$$Dg = \mathbb{R}, Df = (0, +\infty) \Rightarrow \Lambda n - n^2 > 0 \Rightarrow n^2 - \Lambda n < 0 \rightarrow \frac{0}{+} - \frac{\Lambda}{-} +$$

$$D_{f \circ g} = \{n \in Dg \mid g(n) \in Df\} = (0, \Lambda) = A \quad n \in (0, \Lambda) \leftarrow \text{ج}$$

$$R_{f \circ g} \Rightarrow \log_p -n^2 + \Lambda n \rightarrow S \mid x_{\max} = \frac{-b}{2a} = \frac{-\Lambda}{-2} = \frac{\Lambda}{2} = f$$

$$y_s = -14 + 14 = 14 \Rightarrow \log_2 14 = f$$

$$\Rightarrow R_{f \circ g} = (-\infty, f] = B \Rightarrow A \cap B = (0, f) \rightarrow \{1, 2, 3\}$$

دارای سه عنصر صحیح است.

$$f \circ f(n) = f(f(n)) = 1 - 3f(n) \Rightarrow f(n) = 1 - 3n^2 \quad (9)$$

$$h(n) = n^3 + 2n + 1, g(n) = n - 1 \Rightarrow h(g(n)) = (n-1)^3 + 2(n-1) + 1$$

$$\Rightarrow h(g(n)) = n^3 - 3n^2 + 3n - 1 + 2n - 2 + 1 = n^3 - 3n^2 + 5n - 2$$

$$\Rightarrow h(g(n)) = f(n) \Rightarrow n^3 - 3n^2 + 5n - 2 = -3n^2 + 1 \Rightarrow n^3 + 5n - 2 = 0$$

$$\Rightarrow n^3 = -5n + 2 \rightarrow \text{این معادله جواب دارد}$$

$$f(n) = n - \frac{f}{n}, g \circ f(n) = a \cdot n^3 + 2n, g(1) = \Lambda \quad (10)$$

$$g(1) = \Lambda \Rightarrow f(n) = 1 \Rightarrow n - \frac{f}{n} = \frac{n^2 - f}{n} = 1 \Rightarrow n^2 - 1n - f = 0$$

$$\Rightarrow (n+1)(n-1) = 0 \Rightarrow n = -1 \text{ و } 1$$

$$\Rightarrow g(f(-1)) = g(1) = a(-1)^3 + 2(-1) = -a - 2 = \Lambda \Rightarrow a = -10$$

$$g(f(1)) = g(1) = 4a + \Lambda = \Lambda \Rightarrow a = 0 \quad \text{غیر قابل قبول زیرا } f \text{ تابع زوج نیست}$$

$$\text{بنابراین } g \circ f(n) \text{ و } g(n) \text{ هر دو تابع زوج نیستند و لذا } a = 0 \text{ را رد می‌کنیم.}$$

$$\Rightarrow n = -1 \text{ و } 1 \Rightarrow n = 1 \text{ و } -1 \quad \text{AFACH} \Rightarrow \underline{a = -10}$$