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تکلیف شماره ۱

پرتیال سمتری - دوازدهم ریاضی A

$$g \circ f(x) = g(f(x)) \rightarrow x \in D_f, f(x) \in D_g$$

سوال ۱

$$f(x) = \sqrt{\log \frac{x-1}{y}} \rightarrow \log \frac{x-1}{y} \geq 0 \rightarrow x-1 \geq y^0 \rightarrow x-1 \geq 1 \rightarrow x \geq 2$$

$$\left. \begin{array}{l} \log \frac{x-1}{y} \geq 0 \\ x-1 \geq 0 \rightarrow x \geq 1 \end{array} \right\} x \geq 2$$

$$f(x) \in D_g \Rightarrow g(x) = \sqrt{-x^2 + 4x - 4} \rightarrow -x^2 + 4x - 4 \geq 0 \rightarrow -(x-2)^2 \geq 0 \rightarrow x=2$$

$$\rightarrow \sqrt{\log \frac{x-1}{y}} = 2 \rightarrow \log \frac{x-1}{y} = 4 \rightarrow x-1 = y^4 \rightarrow x-1 = 16 \rightarrow x=17$$

$$\rightarrow D_{g \circ f(x)} = \{17\}$$

$$f \circ g(x) = g \circ f(x) \quad \frac{f(x) = 2x-5}{g(x) = x^2-4x+1} \rightarrow y(x^2-4x+1)-5 = (2x-5)^2 - y(2x-5) + 1$$

سوال ۲

$$\rightarrow yx^2 - 4yx + 11 = 4x^2 - 4yx + 11 \rightarrow yx^2 - 4yx + 11 = 0 \quad x = \frac{4y \pm \sqrt{16y^2 - 4(y^2 - 11)}}{2y}$$

$$= \frac{4y \pm \sqrt{10y^2 + 44}}{2y} \rightarrow a_1 = \frac{4y + \sqrt{10y^2 + 44}}{2y}, a_2 = \frac{4y - \sqrt{10y^2 + 44}}{2y}$$

$$= \frac{400 + 104 + 40\sqrt{104}}{14} + \frac{400 + 104 - 40\sqrt{104}}{14} = \frac{1004}{14} = 71.71$$

$$g(f(x)) = 5x^2 + 11, f(x) = 2x \rightarrow g(2x) = 5(2x)^2 + 11$$

سوال ۳

$$\rightarrow g(x) = \frac{5}{4}x^2 + 11 \rightarrow g(x-1) = \frac{5}{4}(x-1)^2 + 11 = \frac{5}{4}x^2 + \frac{5}{2}x - \frac{5}{4} + 11$$

$$= \frac{5}{4}x^2 - \frac{5}{2}x + \frac{39}{4} \rightarrow \min \left| \frac{b}{2a} = \frac{-\frac{5}{2}}{\frac{5}{2}} = -1 \right|$$

$$f(x) = 2x-5, f \circ g(x) = 2x^2-4x-1 \rightarrow y(g(x))-5 = 2x^2-4x-1$$

سوال ۴

$$\rightarrow y(g(x)) = 2x^2-4x-1 \rightarrow g(x) = x^2-2x+1$$

$$\Rightarrow g \circ f(x) = g(f(x)) = g(2x-5) = (2x-5)^2 - 2(2x-5) + 1 = 4x^2 - 14x - 4x + 1 + 1$$

$$= 4x^2 - 18x + 2$$

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سوال ٥) : $f(x) = g(x) = -x + 1 \rightarrow g(x) = f(x) + x - 1 \rightarrow f(g(x)) = f(f(x) + x - 1) = f(x) + x - 1$

$f(g(x)) = f(x) + x - 1$

$\Rightarrow f(x) + x - 1 = f(x) + x - 1 \rightarrow f(x) = x + f \xrightarrow{f-g: -x+1} g(x) = x - 1$

$f(g(x)) = f(x - 1) + f = x + f \xrightarrow{f-g: -x+1} x + f - x + 1 = 0 \rightarrow \boxed{f = -1}$

سوال ٦) $f(x) = 9^x$ $\left\{ \begin{array}{l} f \circ g(x) = f(\log_9 x) = 9^{\log_9 x} = x \\ g \circ f(x) = g(9^x) = \log_9 9^x = x \end{array} \right. \Rightarrow \boxed{f \circ g = g \circ f = \text{id}}$

$f \circ g(x) \leq g \circ f(x) \rightarrow x^y \leq yx \rightarrow x^y - yx \leq 0 \rightarrow x(x - y) \leq 0$

$\Rightarrow \boxed{\{0, 1\}} \Rightarrow \boxed{\text{جواب: } \{0, 1\}}$

سوال ٧) $f\left(\frac{x^y - y}{x}\right) = x^y + x - f - \frac{y}{x} + \frac{f}{x^y} \rightarrow f(x - \frac{y}{x}) = x - \frac{y}{x} + (x^y + \frac{f}{x^y} - f) + \frac{f}{x^y}$

$\Rightarrow f(x - \frac{y}{x}) = (x - \frac{y}{x}) + (x - \frac{y}{x})^y \xrightarrow{x - \frac{y}{x} = t} f(t) = t + t^y \rightarrow \boxed{f(x) = x + x^y}$

سوال ٨) $f(x) = \log_9 x$ $\left\{ \begin{array}{l} f \circ g = f(g(x)) = f(1x - x^y) = \log_9 (1x - x^y) \\ g(x) = 1x - x^y \end{array} \right.$

$1x - x^y > 0 \rightarrow x(1 - x^y) > 0 \rightarrow \frac{0}{-1+1} \rightarrow D_f = (0, 1)$

$\rightarrow R_f = (-\infty, 1) \rightarrow A \cap B = \{1, 2, 3, 4, 5\} \Rightarrow \boxed{\text{جواب: } \{1, 2, 3, 4, 5\}}$

سوال ٩) $f \circ f(x) = 1 - x^y f^y(x) \rightarrow f(f(x)) = 1 - x^y f^y(x) \rightarrow f(x) = 1 - x^y$

$g(x) = x - 1$ $\rightarrow h(x) = x^y + yx + 1 \rightarrow h(g(x)) = f(x)$

$\rightarrow (x - 1)^y + y(x - 1) + 1 = x^y - yx^y + yx - 1 + yx - y + 1 = x^y - yx^y + 2yx - y = 1 - x^y$

$\Rightarrow x^y + 2yx - y = 0 \rightarrow x^y = y - 2yx \rightarrow \boxed{\text{جواب: } x^y = y - 2yx}$

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$$f(x) = x - \frac{f}{x}, \quad g \circ f(x) = ax^w + vx, \quad g(v) = 1$$

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$$\rightarrow \underbrace{g(f(x))}_w = 1 \rightarrow f(x) = v \rightarrow x - \frac{f}{x} = v \xrightarrow{\times x} x^2 - f - vx = 0 \begin{cases} x = -1 \\ x = f \end{cases}$$

$$g(f(x)) = a \left(\underbrace{x - \frac{f}{x}}_v \right)^w + v \left(x - \frac{f}{x} \right) = 1 \rightarrow vva + v = 1 \rightarrow a = \frac{v}{v^2}$$