آروین فسودیان

1. $gof(x) \longrightarrow x\in D_f \ ,\ f(x)\in D_g$

$D_g: \quad -x^2+4x-4\ge 0 \longrightarrow -(x-2)^2\ge 0 \longrightarrow x=2 \longrightarrow f(x)=2$

$\longrightarrow \log_2^{x-1}=4 \qquad x-1=16 \qquad \boxed{x=17} \longrightarrow D_{fog}=\{17\}$

2. $fog=2(x^2-3x+8)-5=2x^2-6x+16-5=2x^2-6x+11$

$gof=(2x-5)^2-3(2x-5)+8=4x^2-20x+25-6x+15+8=4x^2-26x+48$

$2x^2-6x+11=4x^2-26x+48 \longrightarrow 2x^2-20x+37=0$

$\alpha^2+\beta^2=S^2-2P=100-2(\frac{37}{2})=\boxed{63}$

3. $gof(x)=5x^2+11 \longrightarrow g(f(x))=5x^2+11 \quad \frac{5}{4}(2x)^2+11=g(x)$

$g(x-7)=\frac{5}{4}(x-7)^2+11 \xrightarrow{\text{کمترین مقدار}} \overset{7}{11} \longrightarrow \boxed{11}$

4. $f(x)=3x-4 \qquad fog=3x^2-6x-1 \Leftrightarrow f(g(x))=$

$gof(x)=? \qquad f(g(x))=3(g(x))-4=3x^2-6x-1$

$g(x)=\frac{3x^2-6x+3}{3}=x^2-2x+1 \longrightarrow$

$\longrightarrow g(x)=(x-1)^2 \qquad gof(x)=(3x-4-1)^2=(3x-5)^2=9x^2-30x+25$

5. $fog=-x+5 \qquad f(x)-g(x)=-x+5 \longrightarrow 3f(x)-3g(x)=-3x+15$

$2g(x)-3f(x)=-14 \longrightarrow -3f(x)+2g(x)=-14$

$6x-2-3f(x)=-14 \qquad -g(x)=-3x+1 \longrightarrow g(x)=3x-1$

$3f(x)=6x+12$

$f(x)=2x+4 \longrightarrow f(g(x))=2(3x-1)+4=6x-2+4=6x+2$

$fog=6x+2=0 \qquad x=-\frac{2}{6} \qquad \boxed{x=-\frac{1}{3}}=a$

6. $f(x)=9^x \qquad g(x)=\log_3^x \qquad fog(x)\le gof(x)$

$9^{\log_3^x}\le \log_3^{9^x} \longrightarrow x^{\log_3^9}\le x\log_3^9 \qquad x(x-2)\le 0 \qquad \begin{array}{ccccc} & 0 & & 2 & \\ + & | & - & | & + \end{array}$

$\longrightarrow x^2\le 2x \longrightarrow x^2-2x\le 0$

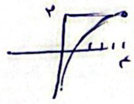
$\longrightarrow x=[0 \quad 2]$

(۷) $f(x-\frac{2}{x}) = (x-\frac{2}{x})^2 + (x-\frac{2}{x}) \rightarrow f(x) = x^2 + x$

(۸) $f(x) = \log_2 x \quad g(x) = 8x - x^2 \quad fog(x) = D_f , R_f \quad A \cap B$

$fog = \log_2^{8x-x^2} \longrightarrow 8x - x^2 > 0 \quad x(8-x) > 0 \quad \frac{\quad 0 \quad 8 \quad}{-\ +\ +\ -} \quad D_f = (0 \quad 8)$

$\max\ 8x - x^2 : \frac{-8}{2} = 4 \quad \boxed{\log_2 16 = 4}$



$\longrightarrow R_f = (-\infty \quad 4)$

$R_f \cap D_f = (0 \quad 4)$

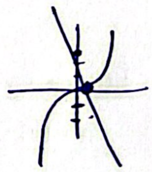
(۹) $fof = 1 - 3f^2(x) \quad g(x) = x - 1 \quad h(x) = x^3 + 3x + 1$

$f(f(x)) = 1 - 3f^2(x)$

$\rightarrow f(x) = 1 - 3x^2$

$(x-1)^3 + 2(x-1) + \cancel{1} = 1 - 3x^2$

$x^3 - \cancel{3x^2} + 3x - 1 + 2x - 2 = \cancel{-3x^2} \quad x^3 + 5x - 3 = 0 \quad x^3 = 3 - 5x$



1 تقاطع دارد

(۱۰) $f(x) = x - \frac{4}{x} \quad gof = ax^3 + 2x \quad g(3) = 8$

$g(f(x)) = ax^3 + 2x$

$f(x) = 3 \quad x - \frac{4}{x} = 3 \quad x^2 - 3x - 4 = 0 \quad (x-4)(x+1) = 0 \quad \rightarrow x = -1, \ x = 4$

$g(f(4)) = 64a + 8 \quad g(\underbrace{f(-1)}_{3}) = -a - 2 \quad -a - 2 = 64a + 8 \quad 65a = -10 \quad a = \frac{-10}{65} = \boxed{\frac{-2}{13}}$