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در متناهیات - در ادامه - تعیین شده است

$$f(x) = \sqrt{\log_v^{x-1}}, \quad g(x) = \sqrt{-x^2 + 4x - 4}$$

$$D_f: \begin{cases} x-1 > 0 \Rightarrow x > 1 \\ \log_v^{x-1} \geq 0 \Rightarrow x-1 \geq 1 \Rightarrow x \geq 2 \end{cases} \Rightarrow x \geq 2$$

$$D_{g \circ f} = \{x \in D_f : f(x) \in D_g\}$$

$$\downarrow \quad \downarrow$$

$$x \in [2, +\infty) \cap \sqrt{\log_v^{x-1}} = 2$$

$$D_g: -x^2 + 4x - 4 \geq 0 \Rightarrow x = 2$$

$$\Rightarrow D_{g \circ f} = \{2\} \quad D_{g \circ f} = \{2\}$$

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$$f(x) = 2x - 5, \quad g(x) = x^2 - 3x + 1$$

$$f \circ g(x) = 2(x^2 - 3x + 1) - 5 = 2x^2 - 4x - 3$$

$$g \circ f(x) = (2x - 5)^2 - 3(2x - 5) + 1 = 4x^2 - 4x + 25 - 6x + 15 + 1 = 4x^2 - 10x + 41$$

$$f \circ g(x) = g \circ f(x) \Rightarrow 2x^2 - 4x - 3 = 4x^2 - 10x + 41 \Rightarrow 2x^2 - 6x + 44 = 0$$

$$\Delta = \frac{b^2}{a} = \frac{36}{2} = 18, \quad p = \frac{36}{2}$$

$$\alpha^2 + \beta^2 = S^2 - 2P = 18 - 36 = -18$$

مجموع مربعات

$$g \circ f(x) = 5x^2 + 11, \quad f(x) = 2x$$

$$\Rightarrow g(f(x)) = 5x^2 + 11 \xrightarrow{f(x)=2x} g(2x) = 5(2x)^2 + 11 = 20x^2 + 11$$

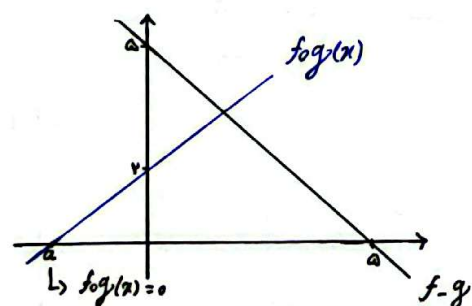
$$g(x) = \frac{5}{4}x^2 + 11 \rightarrow \text{کمترین مقدار} \rightarrow \text{در نقطه ی مسطح تابع می باشد.}$$

همان تابع $y = x^2$ که ناحیه شفافها تغییر کرده و 11 واحد بالا رفته

$$f(x) = 3x - 4, \quad f \circ g(x) = 3x^2 - 4x - 1 \rightarrow f(g(x)) = 3x^2 - 4x - 1$$

$$\Rightarrow 3g(x) - 4 = 3x^2 - 4x - 1 \Rightarrow g(x) = x^2 - \frac{4}{3}x + \frac{1}{3} = (x - \frac{2}{3})^2$$

$$g \circ f(x) = g(f(x)) = (3x - 4)^2 = (3x - 4)^2 = 9x^2 - 24x + 16$$



$$2g(x) = 3f(x) - 14 \Rightarrow g(x) = \frac{3}{2}f(x) - 7$$

$$f(x) - g(x) = f(x) - \frac{3}{2}f(x) + 7 = -\frac{1}{2}f(x) + 7$$

باز هم به هم وصل کردیم

$$-x + 5 = -\frac{1}{2}f(x) + 7 \Rightarrow f(x) = 2x + 4, \quad g(x) = 3x - 1$$

$$f \circ g(x) = 2(3x - 1) + 4 = 6x + 2 \xrightarrow{x=a} 4a + 2 = 0$$

$$\Rightarrow a = -\frac{1}{2}$$

$f(x) = 9^x, g(x) = \log_9 x \rightarrow f \circ g(x) \leq g \circ f(x)$ $f \circ g(x) = 9^{\log_9 x} = x^1$ $g \circ f(x) = \log_9 9^x = x$ $\Rightarrow x^1 \leq x \Rightarrow x^1 - x \leq 0 \Rightarrow x(x-1) \leq 0$ $\Rightarrow x \in [0, 1] \rightarrow \log_9 x \text{ باطل}$ $\Rightarrow (0, 1] \rightarrow x \in \frac{1}{9}$	4
$f\left(\frac{x^2-1}{x}\right) = x^2 + x - \frac{1}{x} + \frac{1}{x^2} \Rightarrow f\left(x - \frac{1}{x}\right) = x^2 - \frac{1}{x} + \frac{1}{x^2} + x - \frac{1}{x} \Rightarrow f(x) = x^2 + x$ $(x - \frac{1}{x})^2$	5
$f(x) = \log_2 x, g(x) = 11x - x^2, D_f = x > 0, D_g = \mathbb{R}$ $A = D_{f \circ g} = \{x \in D_g; g(x) \in D_f\} \Rightarrow x \in (0 \cap (1, 11)) \Rightarrow x \in (1, 11) = A$ $\Rightarrow x \in (1, 11)$ $B = D_{g \circ f} \rightarrow f \circ g(x) = \log_2 (11x - x^2) \rightarrow (-\infty, 11] \Rightarrow R_{f \circ g} = B = (-\infty, 11]$ $\Rightarrow A \cap B = (1, 11]$	6
$f(f(x)) = 1 - 3f(x) \Rightarrow f(x) = 1 - 3x^2$ $g(x) = x - 1$ $h(x) = x^3 + 2x + 1$ $h(g(x)) = (x-1)^3 + 2(x-1) + 1 = x^3 - 3x^2 + 3x - 1 + 2x - 2 + 1 = x^3 - 3x^2 + 5x - 2$ $h(g(x)) = f(x) \Rightarrow x^3 - 3x^2 + 5x - 2 = -3x^2 + 1 \Rightarrow x^3 + 5x - 3 = 0$ $\Rightarrow x^3 + 5x - 3 = 0$	7
$f(x) = x - \frac{1}{x}, g \circ f(x) = ax^3 + 2x, g(w) = 1$ $g(f(x)) = ax^3 + 2x \Rightarrow g\left(x - \frac{1}{x}\right) = ax^3 + 2x \xrightarrow{x=-1} g(w) = -a - 2 \xrightarrow{g(w)=1} -a - 2 = 1$ $\Rightarrow a = -3$ $\rightarrow x = f \rightarrow 4a + 1 = 1 \rightarrow a = 0$	8