۱

$$g(f(x)) \longrightarrow \{x \in D_f \ / \ f(x) \in D_g\}$$

$$D_{f(x)} = \sqrt{\log_2^{x-1}} \longrightarrow \begin{cases} x-1>0 \to x>1 \\ \log_2^{x-1} \geq 0 \to x-1 \geq 1 \to x \geq 2 \end{cases} \xrightarrow{\text{اشتراک}} [2, +\infty)$$

$$\sqrt{\log_2^{x-1}} \longrightarrow \log_2^{x-1} = 4 \to x-1 = 16 \to \boxed{x = 17}$$

$$D_{g(x)} = \sqrt{-x^2+4x-4} \to \sqrt{-(x-2)^2} \longrightarrow x-2=0 \to x=2$$

$$D_g = \{2\}$$

۲

$$f(g(x)) = g(f(x))$$

$$f(x^2-3x+8) = g(2x-5)$$

$$2(x^2-3x+8) - 5 = (2x-5)^2 - 3(2x-5) + 8$$

$$\cancel{2x^2} - 6x + 16 - 5 = \cancel{4x^2} + 25 - 20x - 6x + 15 + 8$$

$$2x^2 - 20x = 25+15+8-11 \Longrightarrow 2x^2 - 20x + 37 = 0 \longrightarrow \alpha^2+\beta^2$$

$$\alpha^2+\beta^2 \to S^2 - 2P \to 100 - 37 = 63$$

$$S = 10 \quad P = \frac{37}{2}$$

۳

$$g(\underbrace{f(x)}_{2x}) = 5x^2+11 \longrightarrow g(2x) = \underbrace{5x^2+11}$$

$$\xrightarrow{\text{طول رأس}} \frac{-b}{2a} = 0$$

$$1 \xrightarrow{\text{جایگذاری}} \boxed{11} \ \text{کمترین مقدار}$$

۴

$$f(x) = 3x - 4$$

$$f(g(x)) = 3x^2 - 6x - 1 \longrightarrow 3g(x) - 4 = 3x^2 - 6x - 1$$

$$\longrightarrow 3g(x) = 3x^2 - 6x + 3 \longrightarrow g(x) = x^2 - 2x + 1$$

$$g(f(x)) \longrightarrow g(3x-4) \longrightarrow (3x-4)^2 - 2(3x-4) + 1$$

$$9x^2 + 16 - 24x - 6x + 8 + 1 \longrightarrow 9x^2 - 30x + 25 = g(f(x))$$

۵

$$f - g = -x + 5 \longrightarrow \begin{aligned} 3f(x) - 3g(x) &= -3x + 15 \\ -3f(x) + 2g(x) &= -14 \end{aligned}$$

$$2g(x) - 3f(x) = -14$$

$$f(x) = 2x + 4$$

$$-g(x) = -3x + 1 \longrightarrow g(x) = 3x - 1$$

$$fog(x) = 2(3x-1) + 4 = 6x + 2 \longrightarrow 6x + 2 = 0 \longrightarrow x = a = \frac{-2}{6} = -\frac{1}{3}$$

$f(g(x)) \leq g(f(x)) \longrightarrow f(\log_3^x) \leq g(9^x)$

$9^{\log_3^x} \leq \log_3^{9^x} \longrightarrow x^{\log_3^9} \leq x \log_3^9 \longrightarrow x^2 \leq 2x$

$x^2 - 2x \leq 0 \longrightarrow x(x-2) \leq 0$ $\quad$ $\underset{+\quad\quad -\quad\quad +}{0 \quad\quad\quad 2}$ $\longrightarrow [0 \;\; 2]$ 6

$\longrightarrow 0, 1, 1, 2 \Rightarrow$ (گزینه ۳)

$f\left(\frac{x^2-2}{x}\right) = x^2 + x - 4 - \frac{2}{x} + \frac{4}{x^2} \longrightarrow f\left(x - \frac{2}{x}\right) = x^2 + \frac{4}{x^2} - 4 + x - \frac{2}{x} \Rightarrow$

$\underbrace{f\left(x - \frac{2}{x}\right)}_{t} = \left(x - \frac{2}{x}\right)^2 + x - \frac{2}{x} \rightarrow f(t) = t^2 + t \xRightarrow{x=t} f(x) = x^2 + x$

7

$f(g(x)) \rightarrow f(8x - x^2) \longrightarrow \log_2^{8x-x^2}$

دامنه $\rightarrow 8x - x^2 > 0 \longrightarrow 0 > x^2 - 8x \longrightarrow 0 > x(x-8)$ $\quad$ $\underset{+\quad - \quad +}{0 \quad\quad 8}$ $\quad (0 \;\; 8) = A$

$D_{fog} = (0 \quad 8)$ 8

$R\, fog(x) \rightarrow R_f \rightarrow x \in D_{fog} \longrightarrow \log_2^{8x-x^2} = fog$ $\quad$ گزینه ۳ / ۱ و ۲ و ۳

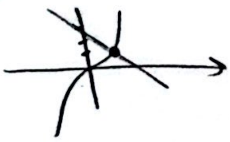
$R(8x - x^2) \longrightarrow \frac{-b}{2a} = 4 \longrightarrow 32 - 16 = 16 \rightarrow R\,g = (0\; 16]$

$\log_2^{x} \longrightarrow 4 \longrightarrow R_{fog} = (-\infty \;\; 4] \cap (0, 8) = (0, 4)$

$f(f(x)) = 1 - 3f^2(x) \longrightarrow f(t) = 1 - 3t^2 \longrightarrow f(x) = 1 - 3x^2$

$h(g(x)) = (x-1)^3 + 2(x-1) + 1 = x^3 - 3x^2 + 3x - 1 + 2x - 2 + 1$

$h(g(x)) = x^3 - 3x^2 + 5x - 2 = 1 - 3x^2 \longrightarrow x^3 + 5x - 3 = 0$ 9

$x^3 = 5x + 3$  $\longrightarrow$ یک نقطه برخورد دارند پس معادله یک جواب دارد.

$g(f(x)) = ax^3 + 2x$ $\quad\quad$ $\left(x - \frac{4}{x} = 3\right) \times x$

$g\left(x - \frac{4}{x}\right) = ax^3 + 2x$ $\quad\quad$ $\hookrightarrow x^2 - 4 = 3x \rightarrow x^2 - 3x - 4 = 0$

$\rightarrow (x+1)(x-4) = 0 \;\begin{smallmatrix} \nearrow x = 4 \\ \searrow x = -1 \end{smallmatrix}$

$\xrightarrow{x=4}\; g(3) = 64a + 8 = 8 \rightarrow 64a = 0 \rightarrow a = 0$ 10

$\xrightarrow{x=-1}\; g(3) = -a - 2 = 8 \longrightarrow -a = 10 \rightarrow a = -10$