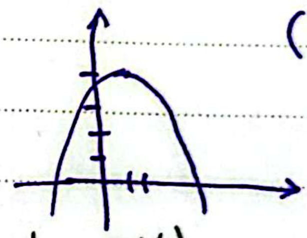
تکلیف ۷ فاطمه نعمانی

(۱ $y=-\frac{1}{3}x^2+\frac{2}{5}x+1 \longrightarrow (4,3)$ ext

$-\frac{1}{3}(x-4)^2+3=y \xrightarrow[y=0]{} -\frac{1}{3}(x-4)^2=-3 \rightarrow (x-4)^2=9$

$\begin{cases} x-4=3 \longrightarrow \boxed{x=7} \\ x-4=-3 \longrightarrow \boxed{x=1} \end{cases}$

(۲ $y=1-2f(3x+4) \longrightarrow A=(\alpha,\beta) \rightarrow \alpha\beta=?$



$f(x) \xrightarrow[\text{اکسترمم}]{} A=(1,4) \xrightarrow[\text{به چپ}]{x+2} (-1,4)$

$\xrightarrow[(\text{انقباض } \frac{1}{3})]{x\times 3} (-\frac{1}{3},4) \xrightarrow{y\times -2} (-\frac{1}{3},-8) \xrightarrow{y+1} (-\frac{1}{3},-7)$

$\alpha.\beta=-\frac{1}{3}\times -7=\boxed{\frac{7}{3}}$ $\quad \alpha \quad \beta$

(۳ $y=3-\sqrt{2x} \xrightarrow[+1]{\text{انتقال به چپ}} 3-\sqrt{2(x+1)}=3-\sqrt{2x+2}$

$\xrightarrow[x]{\text{قرینه نسبت به}} -(3-\sqrt{2x-2})=\sqrt{2x-2}-3 \xrightarrow[5+y]{\text{۵ واحد بالا}} \sqrt{2x+2}-3+5$

$\boxed{y=\sqrt{2x+2}+2}$

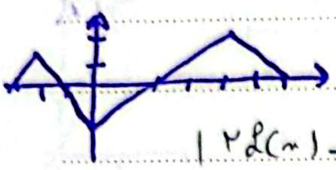
(۴ $D_f=(a,4]$, $D_g=[-1,b]$, $y=f(x-1)+g(x+1) \rightarrow x\in(-2,5)$

1) $f(x-1) \Rightarrow a<x-1\leq 4 \rightarrow a+1<x\leq 5 \rightarrow D_{f(x-1)}\in(a+1,5]$

۲) $g(x+1) \rightarrow -1\leq x+1\leq b \rightarrow -2\leq x\leq b-1 \rightarrow D_{g(x+1)}\in[-2,b-1]$

$\xrightarrow{\cap} (a+1,5]\cap[-2,b-1)=(-2,5)$ $\begin{cases} a+1=-2 \rightarrow \boxed{a=-3} \\ b-1=5 \rightarrow \boxed{b=6} \end{cases}$

۵) $y_1 = f(\frac{x}{2}+1)$; D_f , $y_2 = \sqrt{|2f(x)-3|}$, R_f = ?



$-3 \le x \le 5 \xrightarrow{x \times \frac12} -\frac32 \le \frac{x}{2} \le \frac52 \xrightarrow{y+1} -\frac12 \le \frac{x}{2}+1 \le \frac72$

$|2f(x)-3| \ge 0 \to$ همواره برقرار

$-1 \le f(x) \le 2 \xrightarrow{2f(x)} -2 \le 2f(x) \le 4 \xrightarrow{2f(x)-3} -5 \le 2f(x)-3 \le 1$

$\xrightarrow{||} 0 \le |2f(x)-3| \le 5 \xrightarrow{\sqrt{}} 0 \le \sqrt{|2f(x)-3|} \le \sqrt5$

$\to D_{y_1} = [-\frac12 , \frac72]$, $R_{y_2} = [0 , \sqrt5]$

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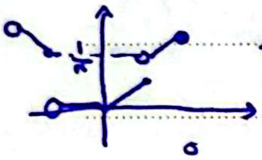
۶) 

$\to y = \frac{x}{a}[ax]$, $b-a=?$

$a<0$

بازه‌بندی $-\frac1a , \frac1a \leftarrow$

$(\frac1a , 0] \in x \to y=0$ / $[0, -\frac1a] \in x \to y = -\frac{x}{a}$

$(-\frac1a , -\frac2a] \in x \to \frac{-2x}{a} \xrightarrow{\text{جایگذاری}} x = -\frac1a \to y = \frac{-2\times -\frac1a}{a} = \frac{2}{a^2}$

$\frac{2}{a^2} = \frac14 \to a = \pm\sqrt{4} \xrightarrow{a<0} a=\sqrt2$, $b = \frac{-2}{a} = \frac{-2}{-\sqrt2} = \sqrt2$

$\to b-a = \sqrt2-(-\sqrt2) = \boxed{2\sqrt2}$

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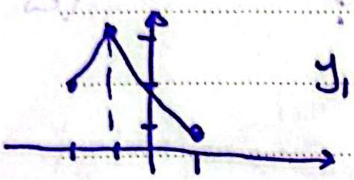
۷) $f(x) = \log_2^x \xrightarrow[\text{؟}]{x+2} \log_2^{x+2} \xrightarrow[x \text{ به}]{\text{قرینه}} -\log_2^{x+2} \xrightarrow{\text{۳ واحد بالا}} -\log_2^{x+2}+3$

$\xrightarrow[y \text{ به}]{\text{قرینه}} -\log_2^{-x+2}+3 \to g(x) = -\log_2^{-x+2}+3 \to g(-14) = -1$, $g(-62) = -3$

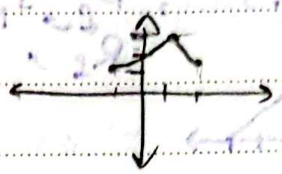
$-1-3 = \boxed{-4}$

$-x\,[-x]$

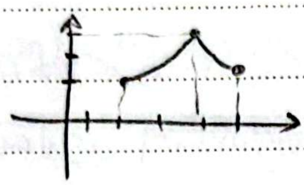
(۸



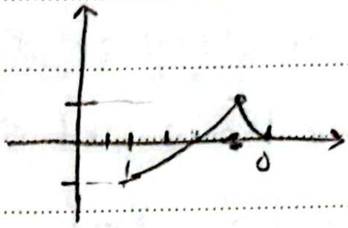
$y_1 = f\,2 - f(3-x)$ $\xrightarrow[\substack{y \text{ به } \\ x^{-1}}]{\text{قرینه}}$



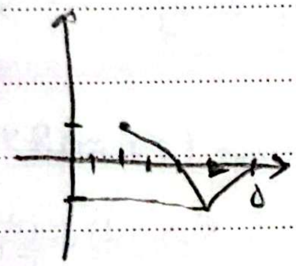
$\xrightarrow{x-3}$



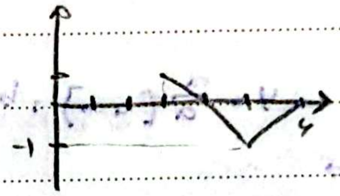
$\xrightarrow{y-2}$



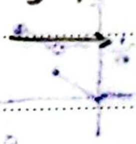
$\xrightarrow{y_{x^{-1}}}$

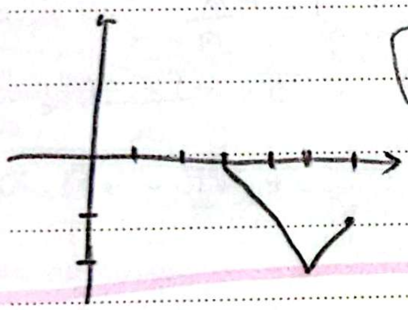


$\xrightarrow{x-1}$



$\xrightarrow{y-1}$





$\boxed{f(x-1)-1=y}$

عوامل کمتر

(۹

$f(x) = \left|\frac{x+1}{2x+1}\right| \xrightarrow[\text{به راست}]{x-2} \left|\frac{(x-2)+1}{2(x-2)+1}\right| = \left|\frac{x-1}{2x-3}\right|$

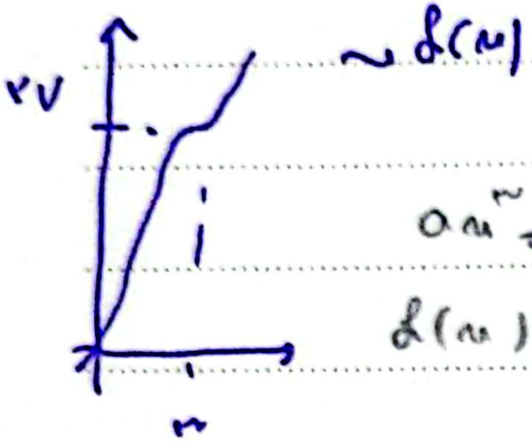
$\xrightarrow{y+1} \left|\frac{x-1}{2x-3}\right|+1 = g(x) \to x+g(x)<0 \to x+\left|\frac{x-1}{2x-3}\right|+1<0$

$1 \qquad \frac{3}{2}$

$x+\frac{x-1}{2x-3}+1<0$	$x+\frac{1-x}{2x-3}+1<0$	$2x-3 \to \frac{2x^2-4}{2x-3}<0$
$\frac{2x^2-3x+x-1+2x-3}{2x-3}<0$	$\frac{2x^2-3x+1-x+2x-3}{2x-3}<0$	$-\sqrt{2} \quad \sqrt{2} \quad \frac{3}{2}$: $- \mid + \mid - \mid +$, $x \geq \frac{3}{2}$
$\frac{2x^2-4}{2x-3}<0 \to$ $-\sqrt{2} \quad \sqrt{2} \quad \frac{3}{2}$: $- \mid + \mid - \mid +$	$\frac{2x^2-2x-2}{2x-3}<0 \to$ $\frac{1-\sqrt5}{2} \quad \frac{1+\sqrt5}{2} \quad \frac{3}{2}$: $- \mid + \mid - \mid +$	$\xrightarrow{\cap} \varnothing$
$\xrightarrow[x<1]{\cap} (-\infty,-\sqrt{2}]$	$1<x<\frac{3}{2} \xrightarrow{\cap}$ $\left[\frac{1+\sqrt5}{2},\frac{3}{2}\right)$	

$\xrightarrow{\cup} \boxed{(-\infty,-\sqrt{2}] \cup \left[\frac{1+\sqrt5}{2},\frac{3}{2}\right)}$

(۱۰۸



خودرا از $(۰,۰)$ گذشته ۷ ⟵

$ax^۳ = f(x) \to x=۲ \to ۸a = ۲۷ \to a = \frac{۲۷}{۸}$

$f(x) = \frac{۲۷}{۸}x^۳ \to \sqrt{۲۸ f(x) - f^۲(x)} = g(x) \to$

$g(x) = \sqrt{۲۸\times\frac{۲۷}{۸}x^۳ - (\frac{۲۷}{۸}x^۳)^۲} \to ۷\times۲۸\, x^۳ - (\frac{۲۷}{۸})^۲ x^۶ \geq ۰$

$\to x^۳\left(۷\times۲۷ - (\frac{۲۷}{۸})^۲ x^۳\right) \geq ۰ \to$

	۰		$\frac{۴}{۳}\sqrt{۷}$	
$-$		$+$		$-$

$\to \boxed{D_f = [۰, \frac{۴}{۳}\sqrt{۷}]}$