هلنا محمودی ، دوازدهم تجربی ، تکلیف شماره ۴

۱:

الف) $(x-2[\frac{x}{2}])^2 \to (2(\frac{x}{2}-[\frac{x}{2}]))^2 \xrightarrow{\square-[\square]\in[0,1)}$

$0\le \frac{x}{2}-[\frac{x}{2}]<1 \xrightarrow{\times 2} 0\le 2(\frac{x}{2}-[\frac{x}{2}])<2 \xrightarrow{\text{به توان ۲}} 0\le y<4$

$\Rightarrow \boxed{R_f=[0,4)}$ ✓

ب) $-\sin x+\log y-4=0 \Rightarrow \log y=\sin x+4 \Rightarrow -1\le \sin x\le 1 \Rightarrow \quad 3\le \sin x+4\le 5$

$\Rightarrow 3\le \log y\le 5 \Rightarrow \boxed{10^3\le y\le 10^5}$ ✓

۲:

$f(x)=\sqrt{\frac{x^2-4}{x^2-9}} \Rightarrow y^2=\frac{x^2-4}{x^2-9} \Rightarrow x^2y^2-9y^2=x^2-4 \Rightarrow x^2y^2-x^2=9y^2-4$

$\Rightarrow x^2(y^2-1)=9y^2-4 \Rightarrow x^2=\frac{9y^2-4}{y^2-1} \Rightarrow x=\pm\sqrt{\frac{9y^2-4}{y^2-1}} \Rightarrow \frac{9y^2-4}{y^2-1}\ge 0 \xrightarrow[y=\pm 1]{y=\pm\frac{2}{3}}$

	-1		$-\frac{2}{3}$		$\frac{2}{3}$		1	
$+$		$-$	0	$+$	0	$-$		$+$

$y\ge 0$

$\Rightarrow [0,\frac{2}{3}]\cup(1,+\infty) \to a+b+c=0+\frac{2}{3}+1=\frac{5}{3}$ ✓

۳:

الف) $f(x)=x+\frac{1}{x}+2\sqrt{x+\frac{1}{x}} \Rightarrow (\sqrt{x+\frac{1}{x}}+1)^2-1$

$\xrightarrow{\text{گذاشتن مقدار}} (\sqrt{2}+1)^2-1 \Rightarrow 2+\cancel{1}+2\sqrt{2}-\cancel{1}=2\sqrt{2}+2 \Rightarrow \boxed{[2+2\sqrt{2},+\infty)}$ ✓

ب) $y=(3-\sin x)(1+\sin x)$

$\hookrightarrow 3+3\sin x-\sin x-\sin^2 x$

$\Rightarrow -\sin^2 x+\underline{2\sin x}+3 \Rightarrow \square \xrightarrow{\max} 1 \Rightarrow -1+2+3=4$

$\square \xrightarrow{\min} -1 \Rightarrow -1-2+3=0 \Rightarrow \boxed{R=[0,4]}$ ✓

$\frac{-b}{2a} \to \frac{-2}{-2}=1 \Rightarrow -1+2+3=4$

۴: اعداد طبیعی یک رقمی: $\{1,2,3,4,5,6,7,8,9\}$

$f(x)=-\frac{1}{3}x+10$

$\Rightarrow x=1 \to y=10-\frac{1}{3}=\frac{29}{3}$

$\Rightarrow x=2 \to y=10-\frac{2}{3}=\frac{28}{3}$

$\Rightarrow x=3 \to y=10-1=9$

$\Rightarrow x=4 \to y=10-\frac{4}{3}=\frac{26}{3}$

$\Rightarrow x=5 \to y=10-\frac{5}{3}=\frac{25}{3}$

$\Rightarrow x=6 \to y=10-\frac{6}{3}=\frac{24}{3}$

$\Rightarrow x=7 \to y=10-\frac{7}{3}=\frac{23}{3}$

$x=8 \to y=10-\frac{8}{3}=\frac{22}{3}$

$x=9 \to y=10-3=7$

$\xrightarrow[\text{مجموع برد تابع}]{\text{مجموع همه‌ی این اعداد}} 75$ ✓

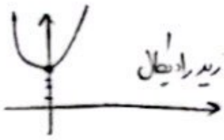
:۵

$$f(x) = \sqrt{ax^2+bx+c} \xrightarrow[\text{باید خطا باشد و نمی‌تواند کلی باشد}]{\text{با توجه به } [2,+\infty)} \underline{a=0} \quad f(3)=1 \Rightarrow 3b+c=1$$

تعیین علامت آن به این صورت است

$$\Rightarrow \begin{array}{c} 2 \\ \hline - \ \phi \ + \end{array} \Rightarrow f(2)=0 \Rightarrow 2b+c=0 \xrightarrow{-2b-c=0} \Rightarrow \underline{b=1} \quad 3(1)+c=1 \Rightarrow \underline{c=-2}$$

$$\Rightarrow f(x) = \sqrt{bx^2+ax-2c} \Rightarrow \sqrt{x^2+4} \Rightarrow \text{زیر رادیکال}$$

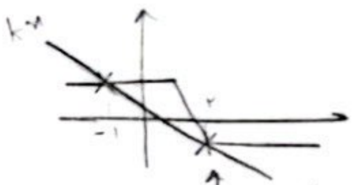


زیر رادیکال $R = [4,+\infty) \xrightarrow[\text{اعمال شود}]{\text{رادیکال باید}} \Rightarrow \boxed{[2,+\infty)}$ ✓

:۶

$$|x-2| - |x-1| = kx \rightarrow \text{جواب ۲)}$$

رسم شکل ←



باید این نقاط را قطع شد

$$\Rightarrow y = kx \xrightarrow{(2,-1)} -1 = 2k \Rightarrow k = -\tfrac{1}{2}$$ ✓

:۷

الف) $y = x[x]$ $\quad [-2,2]$

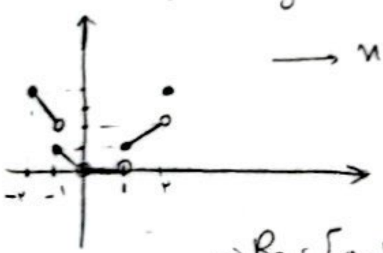
$-2 \le x < -1 \Rightarrow [x] = -2 \Rightarrow y = -2x$

$-1 \le x < 0 \Rightarrow [x] = -1 \Rightarrow y = -x$

$0 \le x < 1 \Rightarrow [x] = 0 \Rightarrow y = 0$

$1 \le x < 2 \Rightarrow [x] = 1 \Rightarrow y = x$

$\longrightarrow x = 2 \rightarrow y = 4$



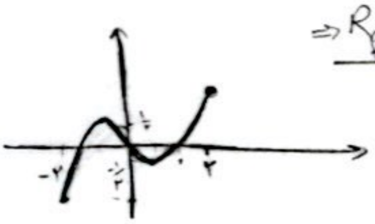
$\Rightarrow R_f = [0,4] - \{2\}$

ب) $y = x|x| - x$ $\quad [-2,2]$

$x < 0 \Rightarrow |x| = -x \Rightarrow -x^2 - x$

$x > 0 \Rightarrow x^2 - x$

$x = 0 \rightarrow y = 0$



$\Rightarrow R_f = \mathbb{R}$

(8

$y = \left(\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x}\right) \times (x)(x+1) \Rightarrow y = \frac{x+x+1+1}{x^2+x} = \frac{2x+2}{x^2+x} = \frac{2(x+1)}{x(x+1)}$

$\xrightarrow{x \neq -1} \frac{2}{x} = y$ اگر $x = -1 \to \frac{2}{-1} = -2$ ، مجانب افقی $\frac{0}{x} = 0 \Rightarrow a = -2,\ b = 0$ با برعکس

$\to a + b = -2 + 0 = -2$ ✓

(9

$f(x) = \frac{x - 4\sqrt{x} + 3}{x - 2\sqrt{x} + 1} = \frac{\cancel{(\sqrt{x}-1)}(\sqrt{x}-3)}{(\sqrt{x}-1)^2} = \frac{\sqrt{x}-3}{\sqrt{x}-1} \xrightarrow[\text{فرم}]{\text{تابع}} \frac{a\square+b}{c\square+d}$

$D_f = \mathbb{R} - \{1\}$

$\Rightarrow Min\ \sqrt{x} = 0 \Rightarrow \frac{0-3}{0-1} = 3$ $\quad Max\ \sqrt{x} = +\infty \quad \frac{+\infty - 3}{+\infty - 1}$ ناچیز $\Rightarrow \frac{+\infty}{+\infty} = 1$

مخرج نمی‌تواند صفر شود پس خارج می‌شود $\quad \sqrt{x} - 1 = 0,\ \sqrt{x} = 1 \Rightarrow x = 1$

$R_f = \boxed{(-\infty, 1] \cup [3, +\infty)}$ ✓

(10

$f(x) = \frac{x^3 + 2x^2 - x - 2}{x^2 - 1} \Rightarrow \frac{x^2(x+2) - (x+2)}{x^2-1} = \frac{\cancel{(x^2-1)}(x+2)}{\cancel{x^2-1}} \Rightarrow y = x + 2$

$D_f = \mathbb{R} - \{\pm 1\}$

$x = 1 \Rightarrow y = 3 \quad x = -1 \Rightarrow y = 1 \quad \Rightarrow 1 + 3 = 4$ ✓