

الف)  $y = (x - 2[\frac{x}{2}])^2 = 4(\frac{x}{2} - [\frac{x}{2}])^2 \rightarrow [0, 1) \xrightarrow{200\%} [0, 1) \cdot \frac{x^2}{4} \rightarrow [0, 4) = R$

ب)  $-\sin x + \log y - t = 0 \rightarrow \log y - t = \sin x \rightarrow -1 \leq \log y - t \leq 1$   
 $\left. \begin{array}{l} 3 \leq \log y \leq 5 \\ y \geq 10^3 \\ y \leq 10^5 \end{array} \right\} \Rightarrow R = [10^3, 10^5]$

$f(x) = \sqrt{\frac{x^2 - 4}{x^2 - 9}} \rightarrow \{y \geq 0 \text{ (سبب اول)}\} \rightsquigarrow y^2 = \frac{x^2 - 4}{x^2 - 9} \quad y^2 x^2 - 4y^2 = x^2 - 4 \rightarrow y^2 x^2 - x^2 = 4y^2 - 4$   
 $x^2 = \frac{4y^2 - 4}{y^2 - 1} \rightarrow x = \pm \sqrt{\frac{4y^2 - 4}{y^2 - 1}} \rightarrow \frac{4y^2 - 4}{y^2 - 1} \geq 0 \quad \frac{-1}{+\frac{1}{9}} - \frac{\frac{4}{9}}{+\frac{1}{9}} + \frac{\frac{4}{9}}{+\frac{1}{9}} - \frac{1}{+\frac{1}{9}} \rightarrow R = [0, \frac{2}{3}] \cup (1, +\infty)$

$[0, \frac{2}{3}] \cup (1, +\infty) = [a, b] \cup (c, +\infty) \rightarrow \begin{cases} a = 0 \\ b = \frac{2}{3} \\ c = 1 \end{cases} \Rightarrow a + b + c = 0 + \frac{2}{3} + 1 = \frac{5}{3}$

الف)  $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}} \xrightarrow{x > 0} \begin{cases} x + \frac{1}{x} \in [2, +\infty) \\ 2\sqrt{x + \frac{1}{x}} \in [2\sqrt{2}, +\infty) \end{cases} \Rightarrow R_f = [2 + 2\sqrt{2}, +\infty)$

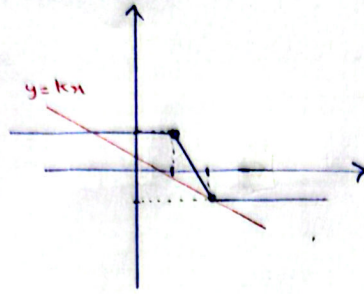
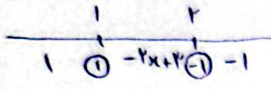
ب)  $y = (3 - \sin x)(1 + \sin x) = -\sin^2 x + 2\sin x + 3$   
 $\left. \begin{array}{l} \sin x = 0 \Rightarrow y = 3 \\ \sin x = 1 \Rightarrow y = 4 \\ \sin x = -1 \Rightarrow y = 2 \end{array} \right\} R = [2, 4]$

$f(x) = -\frac{1}{x} x + 10 \quad D_f = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$   
 $f(1) + f(2) + \dots + f(9) = (9 \times 10) + \left(-\left(\frac{1+2+3+4+\dots+9}{9}\right)\right) = 90 - \left(\frac{45}{9}\right)$   
 $90 - 15 = 75$

$a = 0 \rightarrow f(x) = \sqrt{bx + c}$  چون  $f(x)$  فقط یک ریشه دارد و دامنه آن  $[2, +\infty)$  است داریم  
 $\left. \begin{array}{l} f(2) = 0 \rightarrow 2b + c = 0 \\ f(3) = 1 \rightarrow 3b + c = 1 \end{array} \right\} \rightarrow b = 1, c = -2$   
 $g(x) = \sqrt{bx^2 + ax - 2c} \rightarrow g(x) = \sqrt{x^2 + 4} \quad R_g = [2, +\infty)$



$$|x-2| - |x-1| = kx$$



$$y = kx \xrightarrow{(2, -1)} 2k = -1$$

$$k = -\frac{1}{2}$$

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$$الف) y = x[x]$$

$$-r < x < -1 \quad [x] = -1 \Rightarrow y = -x$$

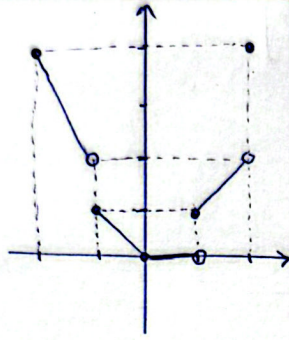
$$-1 < x < 0 \quad [x] = -1 \Rightarrow y = -x$$

$$0 < x < 1 \quad [x] = 0 \Rightarrow y = 0$$

$$1 \leq x < 2 \quad [x] = 1 \Rightarrow y = x$$

$$x = 2 \quad [x] = 2 \Rightarrow y = 2$$

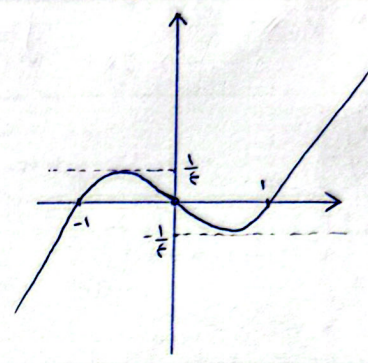
$$R = [0, 2] - \{1\}$$



$$ب) y = x|x| - x$$

$$y = \begin{cases} x^2 - x & x \geq 0 \\ -x^2 - x & x < 0 \end{cases}$$

$$R = \mathbb{R} \quad \text{نقطه‌ای ندارد}$$



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$$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x} = \frac{2x+2}{x^2+x} = \frac{2(x+1)}{x(x+1)} \xrightarrow{x \neq -1} y = \frac{2}{x}$$

$$R = \mathbb{R} - \{-2, 0\} \Rightarrow a = -2, b = 0$$

$$a+b = -2+0 = -2$$

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$$f(x) = \frac{x - \epsilon\sqrt{x} + 3}{x - 2\sqrt{x} + 1} = \frac{(\sqrt{x}-1)(\sqrt{x}-3)}{(\sqrt{x}-1)(\sqrt{x}-1)} \xrightarrow{x \neq 1} f(x) = \frac{\sqrt{x}-3}{\sqrt{x}-1}$$

$$\left. \begin{array}{l} \sqrt{x} = 0 \Rightarrow f(x) = 3 \\ \sqrt{x} = 1 \Rightarrow f(x) = 1 \end{array} \right\}$$

$$R = (-\infty, 1) \cup (3, +\infty) \leftarrow \text{نقطه‌ای ندارد}$$

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$$f(x) = \frac{x^2 + 2x^2 - x - 2}{x^2 - 1} = \frac{(x^2 + 2x^2 - x - 2)(x-1)}{(x+1)(x-1)} = \frac{(x+1)(x+2)(x-1)}{(x+1)(x-1)}$$

$$x \neq \pm 1 \Rightarrow f(x) = x+2 \quad R = \mathbb{R} - \{f(1), f(-1)\} = \mathbb{R} - \{3, 1\}$$

$$1+3 = 4$$

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