نام و نام خانوادگی: رها ارشادی   پایه یازدهم تجربی شماره ۴۰   کلاس ...

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الف) $y=\left(x-2\left[\frac{x}{2}\right]\right)^2$

$\left(2\left(\frac{x}{2}-\left[\frac{x}{2}\right]\right)\right)^2 \quad \xrightarrow{x-[x]\in[0,1)}$

$0\le \frac{x}{2}-\left[\frac{x}{2}\right]<1 \xrightarrow{\times 2} 0\le 2\left(\frac{x}{2}-\left[\frac{x}{2}\right]\right)<2$

$\xrightarrow{\text{توان ۲}} 0\le y<4 \Rightarrow R_f=[0\ \ 4]$

ب) $-\sin x+\log y-4=0$

$\log y=\sin x+4$

$-1\le\sin x\le 1 \to 3\le \sin x+4\le 5$

$\to 3\le\log y\le 5 \to 10^3\le y\le 10^5$

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$f(x)=\sqrt{\frac{x^2-4}{x^2-9}} \xrightarrow{R_f} [a,b]\cup(c\ \ +\infty) \qquad a+b+c=?$

$y=\sqrt{\frac{x^2-4}{x^2-1}} \to y^2=\frac{x^2-4}{x^2-9} \to y^2x^2-9y^2=x^2-4$

$\to y^2x^2=9y^2-4 \Rightarrow x^2(y^2-1)=9y^2-4 \Rightarrow x^2=\frac{9y^2-4}{y^2-1}$

$x=\sqrt{\frac{9y^2-4}{y^2-1}} \xrightarrow{\text{زیر رادیکال}\ \ge 0}$ I $\frac{9y^2-4}{y^2-1}\ge 0 \longrightarrow$

	-1		$-\frac{2}{3}$		$\frac{2}{3}$		1	
$+$		$-$		$+$		$-$		$+$

II $y\ge 0 \xrightarrow[I\cap II]{\text{اشتراک}} \left[0, \frac{2}{3}\right]\cup(1\ ,+\infty) \Rightarrow a+b+c=\boxed{5/3}$ جواب

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الف) $f(x)=x+\frac{1}{x}+2\sqrt{x+\frac{1}{x}}$

$\left(\sqrt{x+\frac{1}{x}}+1\right)^2-1$

$\xrightarrow{\text{کمترین مقدار}} (\sqrt{2}+1)^2-1 \to 2+\cancel{1}+2\sqrt{2}-\cancel{1}$

$\to 2+2\sqrt{2}$

$[2+2\sqrt{2}\ \ +\infty)$

ب) $y=(3-\sin x)(1+\sin x)$

$3+3\sin x-\sin x-\sin^2 x$

$y=-\sin^2 x+2\sin x+3$

$\xrightarrow{max} 1 \Rightarrow -1+2+3=4 \Rightarrow R=[0\ \ 4]$

$\xrightarrow{min} -1 \Rightarrow -1-2+3=0$

$\frac{-b}{2a} \to \frac{-2}{-2}=1 \to -1+2+3=4$

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اعداد طبیعی یک رقمی $\in \{1,2,3,4,5,6,7,8,9\}$

$f(x)=-\frac{1}{3}x+10$

$\xrightarrow{x=1} 10-\frac{1}{3}=\frac{29}{3}$

$\xrightarrow{x=2} 10-\frac{2}{3}=\frac{28}{3}$

$\xrightarrow{x=3} 10-1=9$

$\xrightarrow{x=4} 10-\frac{4}{3}=\frac{26}{3}$

$\xrightarrow{x=5} 10-\frac{5}{3}=\frac{25}{3}$

$\xrightarrow{x=6} 10-2=8$

$\frac{23}{3}=10-\frac{7}{3} \xleftarrow{x=7}$

$\frac{22}{3}=10-\frac{8}{3} \xleftarrow{x=8}$

$7=10-\frac{9}{3} \xleftarrow{x=9}$

$\Rightarrow R_f\left\{\frac{29}{3}, \frac{28}{3}, 9, \frac{26}{3}, \frac{25}{3}, 8, \frac{23}{3}, \frac{22}{3}, 7\right\}$

$\frac{22}{3}+\frac{29}{3}+\frac{28}{3}+9+\frac{26}{3}+\frac{25}{3}+8+\frac{23}{3}+7$

$\Downarrow$

$\boxed{75}$

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$f(x)=\sqrt{\underset{0}{\cancel{a}}x^2+\underset{1}{\cancel{b}}x+\underset{-2}{\cancel{c}}} \xrightarrow{[2+\infty)_{a,b,c}}$

$f(3)=1$

$g(x)=\sqrt{bx^2+ax-2c}=?$

$9a+3b+c=1 \to 9a+3-15a+c=1$

$\boxed{-6a+c=-2}$

$-(4a+2b+c=0) \to 4a+2-10a+c=0$

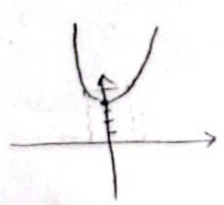
$5a+b=1 \to b=1-5a$

$\boxed{-6a+c=-2}$

$\Rightarrow \begin{cases} 6a+c=-2 \\ -6a+c=-2 \end{cases} \xrightarrow{c=-2} a=0$

$2c=-4 \to \boxed{c=-2}$

$\sqrt{x^2+4}$

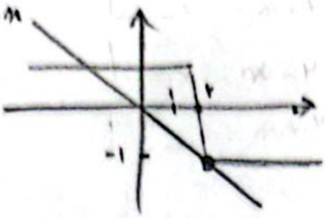


عدد حداقل را در نظر گرفتن

$\to [4\ \ +\infty) \xrightarrow{\text{جذر گرفتن}} [2\ \ +\infty)$

$|x-2| - |x-1| = kx$

شکل را رسم می‌کنیم



$y = kx \xrightarrow{(2,-1)} -1 = 2k \rightarrow \boxed{k = -\frac{1}{2}}$ ۶

الف) $y = x[x]$ ۷

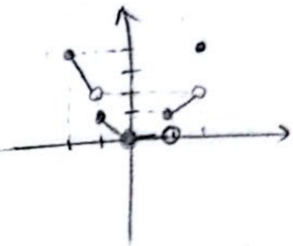
$-2 \leq x < -1 \rightarrow -2x$

$-1 \leq x < 0 \rightarrow -x$

$0 \leq x < 1 \rightarrow 0$

$1 \leq x < 2 \rightarrow x$

$x = 2 \rightarrow 2x$



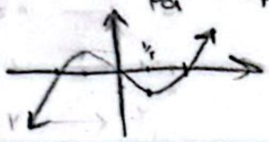
$R = [0, 4] - \{2\}$

ب) $y = x|x| - x$

$$\underbrace{-x^2 - x}_{} \;\overset{0}{|}\; \underbrace{x^2 - x}_{}$$

$\rightarrow -\frac{b}{2a} = \frac{1}{2} \quad y = -\frac{1}{4}$

$\rightarrow -\frac{b}{2a} = -\frac{1}{2} \rightarrow y = \frac{1}{4}$



$R_f = \mathbb{R}$

$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x} \rightarrow R_f = \mathbb{R} - \{a, b\} \rightarrow a + b = ?$ ۸

$y = \frac{x + x + 2}{x^2 + x} = \frac{2x+2}{x^2+x} \rightarrow \frac{2(x+1)}{x(x+1)} \xrightarrow{x \neq -1} \frac{2}{x} = y$

$x = -1 \rightarrow \frac{2}{-1} = -2$ عبارت $= \frac{2}{x} \neq 0$ $a = -2$, $b = 0$

$a + b = -2 + 0 = -2$

$\Rightarrow y = \frac{\sqrt{x} - 3}{\sqrt{x} - 1} \rightarrow y\sqrt{x} - y = \sqrt{x} - 3$ $y = \frac{(\sqrt{x}-3)(\sqrt{x}-1)}{(\sqrt{x}-1)(\sqrt{x}-1)} \xrightarrow[x \neq 1]{\text{اگر}}$ ۹

$y\sqrt{x} - \sqrt{x} = y - 3 \rightarrow \sqrt{x} = \frac{y-3}{y-1} \Rightarrow \frac{y-3}{y-1} \geq 0 \rightarrow$ $\begin{array}{cccc} & 1 & & 3 \\ + & & - & + \end{array}$

$R_f = (-\infty, 1] \cup [3, +\infty)$

$\frac{\cancel{(x+1)}(x^2 + x - 2)}{(x-1)\cancel{(x+1)}} \rightarrow \frac{(x+2)\cancel{(x-1)}}{\cancel{(x-1)}} = x + 2 \xrightarrow{x=1} 3$ ۱۰

$\xrightarrow{x=-1} 1$

$\rightarrow x \neq \pm 1$ $\longrightarrow \boxed{x = 4}$