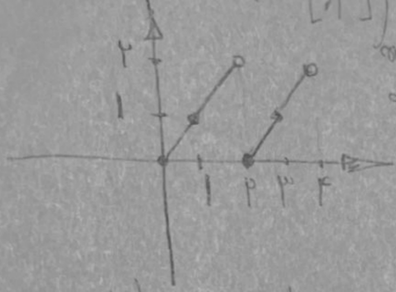
-1

الف) $y=\left(x-2\left[\frac{x}{2}\right]\right)^2 \rightarrow$ $x-2\left[\frac{x}{2}\right] \rightarrow x=1 \rightarrow 1$ $\quad x=3 \rightarrow 1$

$x=2 \rightarrow 0$



$x=0.3 \rightarrow 0.3-0=0.3$ $\quad x=1.2 \rightarrow 1.2-2\left[\frac{1.2}{2}\right]=1.2$

$x=0.8 \rightarrow 0.8$ $\quad x=1.9 \rightarrow 1.9-2\left[\frac{1.9}{2}\right]=1.9$

$x-2\left[\frac{x}{2}\right] \rightarrow [0,2)$ $\quad y=\left(x-2\left[\frac{x}{2}\right]\right)^2 \rightarrow R_f=[0,4)$

ب) $-\sin x+\log y-4=0$ $\quad -1\leqslant \sin x \leqslant 1$

$\log_{10} y = \sin x+4 \rightarrow y>0$ $\quad 10^{+3}\leqslant 10^{\sin x+4}\leqslant 10^5 \Rightarrow 10^3\leqslant y\leqslant 10^5 \Rightarrow R_f=[10^3,10^5]$

-2

$y=\sqrt{\frac{x^2-4}{x^2-9}} \rightarrow y\geqslant 0$ $\quad y^2=\frac{x^2-4}{x^2-9} \Rightarrow x^2y^2-9y^2=x^2-4$ $\quad x^2(y^2-1)=-4+9y^2$

$\Rightarrow x^2=\frac{-4+9y^2}{y^2-1} \Rightarrow x=\sqrt{\frac{-4+9y^2}{y^2-1}}$ $\quad -4+9y^2 \rightarrow \pm\frac{2}{3}$ $\quad y^2-1 \rightarrow \pm 1$

	-1		$-\frac{2}{3}$		$\frac{2}{3}$		1	
$-4+9y^2$	+		+	0	−	0	+	+
y^2-1	+	0	−		−		− 0	+
y	+		−		+		−	+

$x^2 \rightarrow \min=0 \Rightarrow \sqrt{\frac{4}{9}}=\frac{2}{3}$ $\quad y\geqslant 0$ $\Rightarrow R_f=\left[0,\frac{2}{3}\right]\cup(1,+\infty)$ $\quad y\geqslant 0 \Rightarrow R_f=\left[0,\frac{2}{3}\right]\cup(1,+\infty)$

$x^2 \rightarrow \max=+\infty \Rightarrow \sqrt{\frac{\infty+}{\infty+}}=1$

$a=0$, $b=\frac{2}{3}$, $c=1$ $\Rightarrow a+b+c=\frac{5}{3}$

-3

الف) $f(x)=x+\frac{1}{x}+2\sqrt{x+\frac{1}{x}}$ $\quad x+\frac{1}{x}\geqslant 2$, $\quad -x+\frac{1}{x}\leqslant -2$ $\quad x+\frac{1}{x}\geqslant 0 \rightarrow \alpha$ $\Rightarrow 2+2\sqrt{2} \Rightarrow R_f=[2+2\sqrt{2},+\infty)$

ب) $y=(3-\sin x)(1+\sin x)=3+3\sin x-\sin x-\sin^2 x=3+2\sin x-\sin^2 x$ $\quad -1\leqslant \sin x\leqslant 1$

$\rightarrow \max\ 3+2-1=4$

$\min\ 3-2-1=0$ $\Rightarrow R_f=[0,4]$

$-\frac{b}{2a}=-\frac{2}{-2}=1$

-4

$f(x)=-\frac{1}{3}x+10 \rightarrow x=1 \Rightarrow -\frac{29}{3}$ $\quad x=2 \Rightarrow -\frac{28}{3}$ $\quad x=3 \Rightarrow -\frac{27}{3}$ $\cdots$ $x=9 \Rightarrow -\frac{21}{3}$

$1 \rightarrow \frac{9\times -20+(-9+-8+\cdots+-1)}{3}=\frac{-180-45}{3}=\frac{-225}{3}=-75$

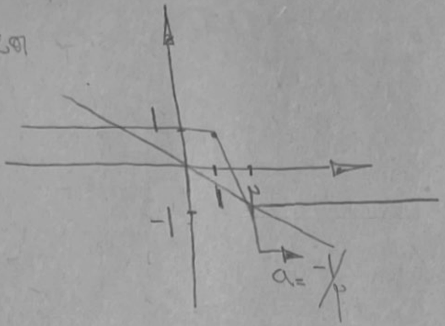
$\rightarrow \frac{9\times 10}{2}=45$

$f(x)=\sqrt{ax^2+bx+c}$ $\quad D_f=[2,+\infty) \Rightarrow a=0$ $\quad \begin{cases} 2b+c=0 \\ 9a+3b+c=1 \end{cases} \Rightarrow \begin{cases} b=1 \\ c=-2 \end{cases}$ $\quad g(x)=\sqrt{x^2+4}$ $\quad \pm 2$ $\quad R_g=(-\infty,-2]\cup[2,+\infty)$

2) $R_f=[2,+\infty) \Rightarrow \frac{-b^2+4ac}{4a}=2$ $\quad 8a=-b^2+4ac$

۶-

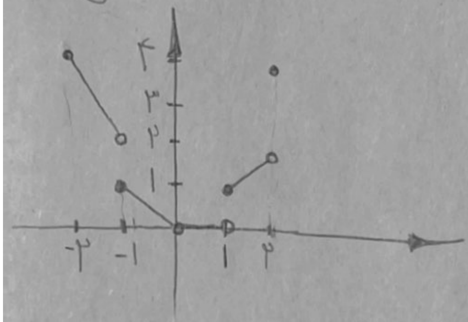
$|x-2|-|x-1| = kx$



$kx \geq -\frac{1}{2}x \qquad k=-\frac{1}{2}$

$a=-\frac{1}{2}$

الف) $y = x[x]$



۷-

$x=0 \Rightarrow 0 \leq x < 1 \rightarrow x\times 0 = 0$

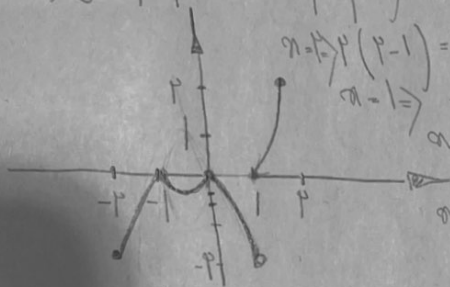
$x=1 \Rightarrow 1 \leq x < 2 \rightarrow x\times 1 = x \qquad 1,9\times 1 = 1,9$

$x=2 \Rightarrow 4$

$x=-1 \Rightarrow -1 \leq x < 0 \rightarrow -1\times x = -x \qquad -0,99\times -1 = 0,99$

$x=-2 \Rightarrow 4 \quad -2 \leq x < -1 \rightarrow -2\times x = -2x \qquad -1,9\times -2 = 3,8 \qquad -1,1\times -2 = +2,2$

ب) $y = x|x| - x = x(|x|-1)$



$x=2 \Rightarrow 2(2-1) = 2$

$x=1 \Rightarrow -1-1=-2$

$x=-1 \Rightarrow -1+1=0$

$x=-\frac{1}{2} \Rightarrow -\frac{1}{2}(+\frac{1}{2})+\frac{1}{2} = -\frac{1}{2}$ $\quad -\frac{1}{4}$

$x=-2 \Rightarrow -4+2=-2$

$x=-\frac{3}{2} \Rightarrow -\frac{3}{2}(+\frac{3}{2})+\frac{3}{2} = -\frac{9}{4}+\frac{6}{4} = -\frac{3}{4}$

$x=0$
$x=\pm 1$

		-1		0		1	
	$-x(+x-1)$ $-x^2+x$		$-x(-x-1)$ $+x^2+x$		$x(-x-1)$ $-x^2-x$		x^2-x
$\lvert x\rvert-1$	$+$		$-$		$-$		$+$
x	$-$		$-$		$+$		$+$
	$-$		$+$		$-$		$+$

۸-

$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x}$

$x \neq -1, 0$

$y = \frac{x+1+x+1}{x^2+x} = \frac{2x+2}{x^2+x} \qquad yx^2+yx = 2x+2 \qquad yx^2-2x+yx-2=0$

$\Rightarrow yx^2+(y-2)x-2=0 \quad \Delta \quad (y-2)^2+8y \geq 0 \quad y^2+4+8y-4y \geq 0 \quad y^2+4y+4 \geq 0$

$\frac{2\cancel{(x+1)}}{x\cancel{(x+1)}} = \frac{2}{x} \quad y=\frac{2}{x} \rightarrow y \neq 0 \qquad (y+2)^2 \geq 0 \quad y \neq -2$

$y \neq \frac{+2}{-1} = -2 \qquad a+b = -2+0 = -2$

۹-

$f(x) = \frac{x-4\sqrt{x}+3}{x-2\sqrt{x}+1}$

$\sqrt{x} = t \quad x \geq 0 \qquad y = \frac{t^2-4t+3}{t^2-2t+1} = \frac{(t-3)\cancel{(t-1)}}{\cancel{(t-1)}(t-1)} = \frac{t-3}{t-1} \rightarrow \frac{y-3}{y-1}$

$t \neq 1$

$yt - y = t-3 \Rightarrow yt - t = y-3 \qquad t(y-1) = y-3 \Rightarrow t = \frac{y-3}{y-1}$

$y \neq 1 \qquad \frac{y-3}{y-1} \geq 0 \qquad + \ 1 \ - \ 3 \ + \quad \Rightarrow R_f = (-\infty, 1) \cup [3, +\infty)$

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۱۰-

$$f(x)=\frac{x^{۳}+۲x^{۲}-x-۲}{x^{۲}-۱}=\frac{x^{۲}(x+۲)-(x+۲)}{x^{۲}-۱}=\frac{\cancel{(x^{۲}-۱)}(x+۲)}{\cancel{x^{۲}-۱}}\Rightarrow \begin{array}{l}۱+۲=۳\\ -۱+۲=۱\end{array}\Rightarrow ۳+۱=۴$$

$$x^{۲}-۱ \longrightarrow x\neq \pm ۱$$